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A
PRACTICAL ARITHMETIC.

BY

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FOR GRAMMAR SCHOOLS.

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PREFACE.

THE object of a text-book on Arithmetic should be to teach the pupil to cipher, — to learn by doing. The shortest and surest road to a knowledge of Arithmetic is by solving problems, not by memorizing rules or by demonstrating propositions. The pupil should be trained to obtain results rapidly and correctly. He should be taught, in questions involving decimal fractions, to limit the answers to the number of decimals required by the nature of the examples, and to avoid all superfluous work. He should not be expected to discover the reason of a process until he fully understands the process; then he should be allowed to state the reason in his own language.

This Arithmetic is not intended for beginners; but it is presumed that pupils will have a thorough knowledge of our "First Steps in Number," and be at least twelve years of age, before entering upon the study of this book.

Decimal fractions are introduced at the beginning of the book. Experience proves that when thus taught they present no difficulty. The difficulty of decimal fractions arises solely from comparing them with common fractions, and is avoided by teaching decimals first. The pupil learns the notation on both sides of the decimal point as easily as on one side; provided the notation on both sides is presented at the same time. Much time is saved by strict adherence to the motto, "Decimal fractions as soon as possible, thoroughly mastered; common fractions postponed as long as possible."

The Metric System in a few years will be in common use, and will supersede other systems, as dollars and cents have superseded pounds, shillings, and pence. Taught immediately after decimal fractions, the system is easily learned. A great number of examples is given to show the simplicity of the system in its application to questions of common occurrence, and to furnish additional practice in operations with decimal fractions. The abbreviations used are such as have been adopted throughout Germany.

Many of the problems are original, but some have been obtained from French, English, and German sources. The examples are intended to convey, incidentally, a great deal of accurate and valuable information; so that the book becomes a book of reference for many physical and mathematical constants.

We gladly acknowledge our obligations to many friends who have improved this work by their advice; and we also give assurance that any suggestions for its further improvement will be thankfully received.

THOMAS HILL.
G. A. WENTWORTH.

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VOCABULARY.

Abstract number. This phrase is employed to designate numbers used without reference to any particular unit, as 8, 10, 21. But *all numbers are in themselves abstract whether the kind of thing numbered is or is not mentioned.*

Addition. The process of combining two or more numbers so as to form a single number.

Aliquot part. A number which is contained an integral number of times in a given number. Thus, 5, 6¼, 12½, 16⅔, are aliquot parts of 100.

Amount. The sum of two or more numbers. In Interest, the sum of principal and interest.

Analysis. The separation of a question into parts, to be examined each by itself.

Antecedent. The first of the two terms named in a ratio.

Area of a surface. The ratio of the surface to another surface assumed as the unit of measure; usually the square of the linear unit.

Arithmetic. The science that treats of numbers and the methods of using them.

Assets. All the property belonging to an estate, individual, or corporation.

Average. The mean of several unequal numbers, so that, if substituted for each, the aggregate would be the same.

Bank. An establishment for the custody, loaning, and exchange of money; and often for the issue of money.

Bank discount. An allowance received by a bank for the loan of money, paid at the time of lending as interest on the sum lent.

Bonds. Written contracts under seal to pay specified sums of money at specified times, issued by national governments, states, cities, and other corporations.

Cancellation. The striking out of a common factor from the dividend and divisor.

Commission. Compensation for the transaction of business, reckoned at some per cent of the money employed in the transaction.

Common denominator. A denominator common to two or more fractions.

Common factor. A factor common to two or more numbers.

Common multiple. A multiple common to two or more numbers.

Complex fraction. A fraction that has a fraction in one or both of its terms.

Composite number. The product of two or more integral factors, each factor being greater than unity.

Compound denominations. Several denominations used to express parts of one quantity.

Compound interest. When the interest due is left unpaid, and considered as an increase made to the principal, the whole interest, accruing in any time, is called compound interest.

Compound fraction. A fraction of another fraction.

Concrete number. A phrase used to denote numbers applied to specified things; as 6 horses, 8 desks.

Consequent. The second of the two terms named in a ratio.

Consignee. The person or firm to whom goods are sent.

Consignor. The person or firm who sends goods to another.

Corporation. An association of individuals authorized by law to transact business as a single person.

Couplet. The two terms of a ratio taken together.

Coupon. A certificate of interest attached to a bond, to be cut off when due and presented for payment.

Creditor. A person or firm to whom money is due.

Cube root. One of the three equal factors of a number.

Customs. Duties or taxes imposed by law on merchandise imported, and sometimes on merchandise exported.

Debtor. A person who owes money to another.

Decimal fractions. Fractions of which only the numerators are written, and the denominators are ten or some power of ten.

Decimal point. A dot placed after the *units'* figure to mark its place.

Decimal system. The common system of numbers founded on their relations to *ten*, *ten tens*, etc.

Denominator. The number which shows into how many equal parts a unit is divided.

Difference. The number which, added to a given number, makes a sum equal to another given number.

Discount. Allowance made for the payment of money before it becomes due. Also, the amount which the market value is *below* the face or nominal value.

Dividend. In division, the given number which is equal to the product of a given factor (called divisor) and required factor (called quotient). In business, the share of profits which belongs to each owner of stock, on his proportion of the capital.

Division. The operation by which, when a product and one of its factors are given, the other factor is found.

Divisor. The number by which a given dividend is to be divided.

Draft. A written order directing one person to pay a specified sum of money to another.

Drawee of a draft. The person to whose order the sum of money named in a draft is to be paid.

Drawer of a draft. The person who signs the draft.

Duty. A sum of money required by government to be paid on the importation, exportation, or consumption of goods.

Equation. A statement that two expressions of number are equal.

Equation of payments. The finding of an average time at which several payments may be justly made.

Exchange. A system of paying debts, due to persons living at a distance, by transmitting drafts instead of money.

Exponent. A small figure placed at the right of a number to show how many times the number is taken as a factor.

Extremes. The first and last terms of a proportion.

Evolution. The process of finding the root of a number.

Factors. The factors of a number are a set of numbers whose product is the given number; they are assumed to be integral, except in the extraction of roots. In commerce, agents employed by merchants to transact business.

Figures. Symbols used to represent numbers in the common system of notation. Also diagrams used to represent geometrical forms.

Firm. The name under which a company transact business.

Fractions. One or more of the equal parts into which the unit is divided.

Grace. An allowance of three days, after the date a note becomes due, within which to pay the note.

Gram. The unit of weight in the metric system.

Greatest common measure. The greatest number which is a common factor of two or more given numbers.

Improper fraction. A fraction whose numerator equals or exceeds the denominator.

Index. A figure written at the left and above the radical sign to show what root of the number under the radical sign is required. A fraction written at the right of a number, of which the numerator shows the required power of that number, and the denominator the required root of that power.

Instalment. A payment in part.

Insurance. A guarantee of a specified sum of money in the event of loss of property by fire, storm at sea, or other disaster; or of loss of life.

Integral number. A number which denotes whole things.

Interest. The sum paid for the use of money.

Involution. The process of finding a power of a number.

Latitude of a point. The angle made by the vertical line at that point with the plane of the equator.

Least common multiple. The least number which is a common multiple of several given numbers.

Liability. A debt, or obligation to pay.

Line. Length without breadth or thickness. The path of a moving point.

Liter. The unit of capacity in the metric system equal in volume to a cube each edge of which is one-tenth of a meter.

Long division. The method of dividing in which the processes are written in full.

Longitude of a point. The angle between two planes supposed to pass through the centre of the earth and to contain, the one the meridian of that point, and the other the standard meridian.

Loss. The excess of the cost price above the selling price.

Maturity of a note. The date at which a note legally becomes due.

Mean proportional. A number which is both the second and third terms of a proportion.

Means. The terms of a proportion between the extremes.

Meter. The unit of length in the metric system.

Minuend. The given number in subtraction which is equal to the sum of another given number called the subtrahend, and a required number called the difference or remainder.

Mixed number. A number that expresses both entire things and parts of things taken together.

Multiple of a number. The product obtained by taking the given number an integral number of times.

Multiplicand. The number to be multiplied by another.

Multiplication. The operation of finding a number bearing the same ratio to the multiplicand which the multiplier bears to unity.

Multiplier. The number by which the multiplicand is multiplied.

Net proceeds. The amount that remains of the money received for property after paying all expenses incurred in disposing of it.

Notation. A system of expressing numbers by symbols.

Note. A written agreement to pay a specified sum of money at a specified time.

Number. The answer to the question, How many?

Numeration. A system of naming numbers.

Obligation. A debt, or liability to pay.

Order of units. A name used to designate the number of things in a group, as *tens*, *hundreds*, *thousands*, etc.

Partial payment. Part payment on a note.

Partnership. An association of two or more persons to carry on business.

Par value. Face or nominal value.

Pendulum. A body suspended by a straight line from a fixed point, and moving freely about that point as a centre.

Percentage. A part of any given number reckoned at some rate per cent.

Period. A group of three figures.

Policy. A written contract of insurance.

Poll tax. A tax levied by the head or poll.

Power. The product of two or more equal factors.

Premium. The sum paid for insurance computed at some rate per cent of the amount insured. Also the excess of market value above par value.

Present worth. The present value of a debt due at some future day.

Prime number. A number which has no integral factors except itself and one.

Principal. The sum of money drawing interest.

Problem. A question to be solved.

Product. The result obtained by multiplying the multiplicand by the multiplier.

Profit. The excess of selling price above cost.

Proof The evidence by which the accuracy of any result is established.

Proper fraction. A fraction, the numerator of which is less than the denominator.

Proportion. A statement that two ratios are equal.

Quantity. The answer to the question, How much?

Quotient. The number sought in division.

Rate per cent. Rate by the hundred.

Ratio. The *relative magnitude* of two numbers or of two quantities.

Reciprocal of a number. *One* divided by that number.

Reduction. The process of changing the *unit* in which a quantity is expressed without changing the *value* of the quantity.

Remainder. The number which, added to the subtrahend, gives a sum equal to the minuend.

Root of a number. One of the equal factors of the number.

Rule. The statement of a prescribed method.

Security. Property used to guarantee the payment of any debt.

Share. One of a certain number of equal parts into which the capital of a company is divided.

Short division. The method of dividing in which the operations of multiplying and subtracting are performed mentally.

Solid. A magnitude which has length, breadth, and thickness.

Solution. The process by which the answer to a question is obtained.

Specific gravity of a substance. The ratio of the weight of a given volume of it to that of an equal volume of water.

Square root. One of two equal factors.

Stock. Capital invested in business.

Subtraction. The process of finding a number which added to one of two given numbers will produce the other.

Sum. The number which results from combining two or more numbers together.

Surd. An indicated root the value of which cannot be exactly expressed in figures.

Surface. That which has only length and breadth.

Thermometer. An instrument for measuring heat.

Unit. A single thing. Also, an arbitrary length, adopted as a standard of measure, in terms of which all measurements are expressed.

Verify. To establish, by experiment, the truth of any statement.

Volume of a solid. The ratio of a solid to an assumed unit of measure; usually a cube of the linear unit.

ARITHMETIC.

CHAPTER I.

NUMBERS.

1. A FUNDAMENTAL IDEA, like that of number, cannot be defined. A number is the simple, direct answer to the question, How many?

2. A system of naming numbers is called **numeration**, and a system of representing numbers by symbols is called **notation.**

The first twelve numbers, formed by adding one thing at a time, are named:

One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve.

These numbers are represented by Arabic figures, or digits, as follows:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

3. The digit 0, called naught, zero, or cipher, is used as a symbol of the words "none" or "nothing." A digit written in the **second** place to the left of another digit signifies **tens.** Thus, 10 is one ten and nothing; 11 is one ten and one; 12 is one ten and two; 23 is two tens and three; etc., etc.

4. The next seven numbers, 13, 14, 15, 16, 17, 18, 19, are the teens: thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.

5. Two tens are called **twenty**; three tens, **thirty**; four, **forty**; five, **fifty**; six, **sixty**; seven, **seventy**; eight, **eighty**; nine, **ninety**.

Thus, 57 is fifty-seven; 63, sixty-three; 89, eighty-nine; etc.

6. Read the numbers:

14, 13, 11, 12, 19, 18, 17, 16, 15, 10, 20, 22, 21, 27, 26, 25, 28, 23, 24;

30, 32, 31, 36, 38, 34, 35, 37, 33, 29, 44, 55, 66, 77, 43, 52, 61, 42, 41, 45, 49, 57, 47, 60, 56;

67, 76, 53, 39, 58, 75, 77, 83, 87, 92, 91, 96, 87, 99, 98, 85, 74, 84, 48, 46, 44, 64, 65, 63.

7. Ten tens are one **hundred**; and hundreds are written in the **third** place toward the left; thus:

100, one hundred; 700, seven hundred; 823, eight hundred twenty-three; 941, nine hundred forty-one; 607, six hundred seven.

8. Read the numbers:

121, 222, 78, 780, 778, 708, 827, 741, 531, 712, 231, 365, 474, 439, 143;

534, 587, 932, 239, 392, 617, 761, 176, 873, 246, 789, 879, 987, 978, 934, 543, 897;

707, 680, 704, 518, 405, 903, 319, 309, 104, 870, 801, 807, 708, 610, 909, 120, 201.

9. Write in figures:

Seventeen, seventy, nineteen, ninety-three, sixteen, sixty-seven, fourteen, forty-five, fifty-nine;

One hundred three, seven hundred nineteen, three hundred three, six hundred eight, six hundred eighteen, six hundred eighty;

Nine hundred ten, five hundred six, eight hundred eighty-eight, four hundred twenty-seven, seven hundred sixty-five, two hundred eleven.

10. Ten hundred make one **thousand**, and thousands are written in the **fourth** place to the left.

Tens of thousands are written in the **fifth** place, **hundreds of thousands** in the **sixth** place.

A comma is usually placed among the figures at the point where, in reading, the word thousand comes.

Thus, 621,453 is 621 thousand, 453; and 700,006 is 7 hundred thousand, six.

11. Read the numbers:

1000; 7000; 7842; 1067; 5043; 8375; 18,757; 75,764; 234,567; 34,561; 123,456;

654,089; 600,897; 607,804; 2753; 704,608; 900,008; 806,042; 709,053; 350,709; 240,682; 118; 537; 3094;

5000; 50,000; 500,005; 682,000; 246,001; 753,110; 703,101; 930; 2020; 200,200; 708,056; 70,856; 870,890.

12. Write in figures:

Eight hundred thousand eight hundred eight; six hundred thousand six; seven hundred thirteen thousand three hundred nine.

Five hundred seventeen thousand six hundred thirteen; seven thousand eight hundred fifty-four; seven hundred eighteen thousand eight hundred eighty-nine.

13. **A thousand thousands** make a **million**; and millions up to 999 millions are written in the **seventh, eighth,** and **ninth** places.

Thus, 728,651,423 is 728 million, 651 thousand, 423.

14. Read the numbers:

21,978,564; 17,756,423; 300,200,100; 707,303,202; 312,406,780; 908,765,487; 2,632,561.

15. Digits are thus divided by commas into **periods** of three places each. Each period contains its own units' place, tens' place, and hundreds' place; but the **unit** in the **first** period, on the right, is **one**; in the **second** period, a **thousand**; in the **third** period, a **million**; and so on.

The **first** period is called the **units'** period; the **second** is the **thousands'** period; the **third** is the **millions'** period. A **fourth** period is the **billions'** period; a billion being a thousand millions; and the **fifth** period the **trillions'** period. The next periods, taken in order, are quadrillions, quintillions, sextillions, septillions, octillions, and so on.

16. We may read the number 1736 either as 1 thousand 736, or as 17 hundred 36. In like manner, we may read a larger number in a still greater variety of ways; thus, 2,736,000,000, may be read, 2 billions, 736 millions; or, 2 thousand 736 millions; or, 27 hundred 36 millions. To put this into the form of a rule:

When numbers are expressed in words, the name of a smaller number following a larger signifies that the two *numbers are to be added; but the name of a smaller number coming before a larger, or before an equal number, signifies that the second is to be multiplied by the first.*

17. Read the numbers:

764,123,897; 40080; 795,013; 103,547,020; 71,003,054; 3,125,476,890; 79,501,346,081; 3,001,574.

18. Write in order the numbers from 1 to 25; from 95 to 115; from 195 to 215; from 985 to 1011; from 9995 to 11111; from 99999 to 100,011.

19. Write in figures, then read from the figures:

One hundred ten; one hundred one; two hundred seven; three hundred seventy; six hundred forty-one; seven hundred thirty-two; eight hundred eight; eight hundred eighty; nine hundred sixty-four; nine hundred ninety-three; one hundred seven; one thousand seven; one thousand ten.

Six million one thousand one; three million five; five million six; four million three hundred thousand three hundred three; seven million six hundred thousand eight hundred twenty-nine; eighty-one thousand ninety-five; seventeen hundred thousand millions; twenty-one thousand thousands; eighty-three million millions.

One hundred ten million two hundred seventy-nine; nineteen trillion four million three hundred three; one quadrillion one hundred twelve trillion three hundred thirty-four million two hundred eleven; ten quintillion two trillion three hundred billion five thousand seven.

CHAPTER II.

DECIMAL FRACTIONS.

20. THOSE things of which we do not naturally ask, How many? but, How much? we endeavor to measure; and we answer the question, How much? by answering, How many measures?

21. The measure of any kind of quantity is readily conceived as divided into ten smaller measures. A dollar, for example, as a measure of value, is divided into ten dimes; each dime into ten cents; each cent into ten mills.

22. Dollars are signified by the mark $ written at the left of the figures.

When part of a dollar is to be written, a full point is written after the dollars, the dimes are written to the right, then the cents and mills.

For example: $17 is seventeen dollars; $18.20 is eighteen dollars, two dimes, or eighteen dollars, twenty cents; $35.875 is 35 dollars, 87 cents, 5 mills; $0.08 is 8 cents.

23. Read as dollars, cents, and mills: $76.375; $163.58; $241.185; $357.34; $12.50; $0.875; $0.125; $1.01; $10.10.

24. A dime is the tenth of a dollar, a cent the hundredth of a dollar, a mill the thousandth of a dollar.

Parts of other measures than those of value may be written in the same way; with tenths, hundredths, etc., to the right of a point. Thus, if we omit the mark $ from $ 5.375, it may stand for 5 quarts, yards, bushels, or any other full measures, and 375 thousandths of another measure.

25. Parts thus written are called *Decimal Fractions.* We write and number to the right of the units' place, precisely as we do to the left, first carefully marking the units' place with a decimal point to its right. Thus, in the figures

9,876,543,210.123,456,789

the full point after 0 shows that 0 stands in the units' place. The 1 to the left is 1 **ten,** the one to the right is 1 **tenth;** the 2 to the left is 2 **hundreds,** the 2 to the right is 2 **hundredths;** the 3 to the left is 3 **thousands,** the 3 to the right is 3 **thousandths;** the 4 to the left is 4 **ten-thousands,** the 4 to the right is 4 **ten-thousandths;** the 5 to the left is 5 **hundred-thousands,** the 5 to the right is 5 **hundred-thousandths;** the 6 to the left is 6 **millions,** the 6 to the right is 6 **millionths;** and so on.

In like manner, the 210 is the units' period; the 543 is the **thousands',** the 123 the **thousandths'** period, etc.; so that the number may be read 9 billions, 876 millions, 543 thousands, 210, and 123 thousandths, 456 millionths, 789 billionths.

26. In reading decimal fractions in this manner, we are obliged, if the right-hand period contains less than three places, to fill the missing places mentally with naughts. For example, .0004 would be read 400 millionths. This is sometimes objectionable (for reasons which will hereafter be plain to the student), and therefore the more usual way of reading is:

Read the decimal precisely as if a whole number, and add the fractional name of the lowest place.

For example, 5.17 is read 5 and 17 hundredths; 5.0017, five and 17 ten-thousandths; 6.0203107, six and 203 thousand 107 ten-millionths.

In either of these ways of reading decimals, the word "and" is distinctly pronounced at the decimal point and carefully omitted in all other places. Thus, one hundred forty-seven means 147; but one hundred *and* forty-seven thousandths means 100.047; and .147 must be read one hundred forty-seven thousandths.

Another ambiguity in this way of reading can be avoided only by a pause; thus, .300 is three hundred . . . thousandths, while .00003 is three . . . hundred-thousandths.

27. To avoid these ambiguities, practical computers introduce the word "decimal" at the place of the point, and then pronounce the digits in succession to the right. Thus, 203.07051 is read two hundred three, decimal, naught, seven, naught, five, one.

28. Read the mixed numbers:

17.23; 18.41; 27.49; 341.07; 1.52; 0.52; .52; .1357; 201.106; 11.111; 13.013; 17,000.017; 6132.0173; .0609; .00613; 26.7; 2.67; .267; .00267; 195.123; 0.83; 0.0087; .00091; 3.1416; 3.14159; 3.14159265.

29. The necessity for putting the decimal point in its right place may be seen on comparing \$312.50, \$31.25, \$3.125, and \$0.3125. *To move the point one place is to multiply or divide by ten.*

30. Write in figures, then read from the figures:

75 hundredths; 8 thousandths; 7 tenths; sixty hundredths; 77 thousandths; 83 ten-thousandths; eight and three ten-

thousandths; six, decimal, naught, naught, one, naught, three; five, decimal, six, naught, seven, three, one;

Nine and 43 millionths; 143 millionths; one hundred and 43 millionths; one hundred forty and three millionths; nine hundred forty-three thousand and nine hundred forty-three thousandths; 722 ten millionths; thirteen, decimal, naught, one, four, six, eight.

31. Point 6753241 in eight different ways, and read each. Point 8957 in eight different ways, prefixing or annexing naughts, and read each.

32. Write in figures:

Three hundred seven; three hundred and seven thousandths; three hundred seven thousandths; three and one hundred seven thousandths; three hundred seven and fourteen thousandths.

33. Read as dollars, cents, and mills:

$24.073; $16.187; $35.625.

Read the same figures as whole numbers and decimal fractions without naming any unit.

34. Write in figures:

81 thousand and 345 thousandths; 37 hundred 41 and 675 thousandths; 4 hundred 13 and 8 hundredths; 96 and 96 thousandths.

Read them as dollars, cents, and mills.

35. Write as money, then read as whole numbers and fractions without naming any unit:

Five and a half dollars; thirteen dollars and twenty-five cents; 83 dollars and 14 cents; 60 dollars and 12 and a half cents; 6 dollars, 9 cents, and 9 tenths of a mill.

REVIEW I.

Numeration is a system of *naming* numbers.

The names of numbers from one to twelve are all different. The names of numbers between twelve and twenty are formed by adding ten to names of numbers less than ten; as, thirteen, fourteen, etc.

Tens are counted like simple units. Thus, two tens, three tens, called twenty, thirty, etc.

Ten tens are called a *hundred,* just as ten units are called a ten. That is, we regard a ten as a *single group*, and call it a *unit of the second order;* a hundred as a *group of ten tens*, and call it a *unit of the third order.*

The names of numbers between any two tens, as from twenty to thirty, are formed by adding to the name of the smaller number of tens the names of numbers less than ten; as, twenty-one, twenty-two, etc.

Hundreds are counted like tens; as, one hundred, two hundred, etc., up to ten hundreds, which is called a *thousand.*

The names of numbers between any two hundreds are formed by adding to the name of the smaller number of hundreds the names of the numbers less than a hundred; as, one hundred one, one hundred two, etc.

A thousand is regarded as a single group of ten hundreds, and is called a *unit of the fourth order.*

Thousands are counted precisely like hundreds, tens, and simple units, up to ten thousands.

The names of numbers between any two thousands are formed by adding to the name of the smaller number of thousands the names of numbers less than a thousand; as, one thousand one, one thousand two, etc.

Ten thousand is regarded as a *unit of the fifth order;* a hundred thousand as a *unit of the sixth order.*

Ten hundred thousand is called a *million*, and is regarded as a *unit of the seventh order;* and so on, always forming from *ten units of the same order one unit of the next higher order.*

It is necessary to observe that we say a *unit of thousands*, a *ten of thousands*, a *hundred of thousands*, just as we say a simple unit, a ten of units, a hundred of units. That is, we regard the thousands as a *second class* of units composed of *three orders*, just as the *first class* of units is composed of *three orders.*

Likewise we regard the million as a *third class* of units, containing the units of millions, the tens of millions, the hundreds of millions.

The following table presents the names and the succession of the units of the different orders and different classes:

Fourth Class. BILLIONS.			Third Class. MILLIONS.			Second Class. THOUSANDS.			First Class. SIMPLE UNITS.		
Twelfth Order. Hundreds of Billions.	Eleventh Order. Tens of Billions.	Tenth Order. Units of Billions.	Ninth Order. Hundreds of Millions.	Eighth Order. Tens of Millions.	Seventh Order. Units of Millions.	Sixth Order. Hundreds of Thousands.	Fifth Order. Tens of Thousands.	Fourth Order. Units of Thousands.	Third Order. Hundreds.	Second Order. Tens.	First Order. Simple Units.

That is, **the common system of numeration** is founded upon the following principle: *Ten units of any order are equal in value to one unit of the next higher order;* and *one unit of any order is equal in value to ten units of the next lower order.*

Decimal fractions are formed by dividing the unit into **ten equal parts,** called *tenths;* the tenth into ten equal parts, called *hundredths;* the hundredth into ten equal parts, called *thousandths;* the thousandth into ten equal parts, called *ten-thousandths*, etc.

By comparing these different decimal fractions, we see that the formation of decimal fractions depends, like that of whole numbers, upon the following principle: *Ten units of any order are equal in value to one unit of the next higher order;* and *one unit of any order is equal in value to ten units of the next lower order.*

Notation is a system of *writing* numbers.

The common system of notation employs ten figures, or digits: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. The first nine figures represent the first nine numbers; the last, which is called zero, or naught, is used to denote the *absence of units* of the order in which it stands. These ten figures express all numbers by the artifice of making the value of each figure increase *tenfold for every place that it is moved to the left.*

To write a number in figures, we write successively the number of units of each *order* from left to right, beginning at the highest order, taking care, if the number contain a decimal fraction, to put a full point at the right of the units' figure, and to supply by zero the units of any order that may be lacking. If the number contain no whole number, we put a *zero in the units' place,* and the decimal point to the right of the zero.

To read a number written in figures, we divide the number into periods of three figures each, from right to left. This done, we begin to read at the last period on the left, and read *as if the figures of that period stood alone*, adding the *name* of the period. Then the next period to the right is read, with the *name* of that period, and so on.

If the number contain a decimal fraction, we first read the whole number; then the decimal as a whole number, taking care to add the *fractional name of the lowest place.*

CHAPTER III.

ADDITION.

36. ADDITION means putting together. The Saint George's cross + is read **plus**, and means that the numbers between which it stands are to be added together.

The result obtained by adding together two or more numbers is called their **sum.**

The sign of equality = stands for the words "equals," or "equal."

Thus, $8 + 1 = 9$ is read, eight plus one equals nine.

37. Add 2 to each number from 0 to 9; add 3 to each number from 3 to 9.

Add 4 to each number from 4 to 9; add 5 to each number from 5 to 9.

Add 6 to each number from 6 to 9; 7 to 7, to 8, and to 9; 8 to 8, and to 9; 9 to 9.

Repeat these additions until thoroughly familiar with them.

38. Name, as fast as you can talk, the even numbers, 2, 4, 6, etc., up to 102.

Name the odd numbers, 1, 3, 5, etc., up to 101.

39. Name every third number, 0, 3, 6, etc., up to 102.

Name every third number, 1, 4, 7, etc., up to 103.

Name every third number, 2, 5, 8, etc., up to 101.

40. Name every alternate even number, 0, 4, 8, 12, etc., up to 100.

Name every alternate even number, 2, 6, 10, etc., up to 102. Name every alternate odd number, 1, 5, 9, etc., up to 101. Do the same, beginning 3, 7, 11, etc., and go to 103.

41. Name every fifth number under 100, beginning 5, 10, 15, etc.; beginning 1, 6, 11; beginning 2, 7, 12; beginning 3, 8, 13; and beginning 4, 9, 14.

42. In like manner, add by sixes up to a number exceeding a hundred, beginning 0, 6, 12; beginning 1, 7, 13; beginning 2, 8, 14; and so on.

43. Add by sevens up to a number exceeding 100, beginning 0, 7, 14; 1, 8, 15; and so on; by eights, beginning 0, 8, 16; 1, 9, 17; and so on; by nines, beginning 0, 9, 18; 1, 10, 19; and so on.*

44. For the addition of numbers in general, the following mode has been found most convenient.

Write the numbers in columns, units under units, tens under tens, tenths under tenths, etc.

Add the digits in the right-hand place; set the units of the sum in that place, but carry the tens mentally to the next place to the left, to be added to the digits there, and so proceed.

The study of three or four examples will make the process understood.

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* The teacher may take ten small cards. On each side of the first write 0; of the second, 1; etc. Shuffle, and dictate the numbers on all but one for the class to add. Subtract the reserved number from 45; the remainder is the sum. With two sets of cards at once, subtract the reserved number from 90. For advanced classes, use cards with larger numbers; and complemental cards, which may be obtained of the publishers, furnishing unlimited examples, with the answers to all.

$$\begin{array}{r} 4128 \\ 3789 \\ 2667 \\ \underline{5821} \\ 16405 \end{array}$$

The following wording, *and no more*, is to be used: 1, 8, 17, **25** (emphasize 5, and write it down while pronouncing it), carry 2; 4, 10, 18, **20**, carry 2; 10, 16, 23, **24**, carry 2; 7, 9, 12, **16**.

$$\begin{array}{rrr} 271.188 & 2.7113 & 314. \\ 51.27 & .51278 & .0561 \\ \underline{1003.684} & \underline{10.03684} & \underline{27.53} \\ 1326.142 & 13.26092 & 341.5861 \end{array}$$

45. Find the following sums:

231 + 764; 341 + 57.8; 430.31 + 58.61; 512.87 + 36.84 + 12.78 + 711.56 + 415.86.

46. Add 1543.1 to 164.7; to 1728; to 402.56; to 1897.3; to 475.34; to 6897.65.

47. Add 1897.3 to 475.34; to 6897.65; to 1728; to 402.56; to 164.7; to .5236; to 2.71828.

48. Find the following sums:

.7854 + 3.1416 + 2.71828; .7854 + 3.1416 + 30,103;
2.71828 + 402.56 + 1897.3; 2.7113 + 27.53 + 341.586.

49. Add 737.87 to each of the following numbers:

111; 1011; 2304; 222; 263; 373; 262.13; 561.2; 32.35; 604.3.

50. Find the five sums:

230.8 + 223 + 2.63 + 373.8 + 56.123; 32.358 + 821.9 + 23.04 + 73.7; 202.3031 + 71.575 + 65.813 + .0078 + 7.377; 653.03 + 65.303 + 6.5033; 939.303 + 65.746 + 8.2794 + 681.28.

51. In this article are given 24 decimal fractions, arbitrarily numbered, for convenience in referring to them.

No.	Decimal.	No.	Decimal.
1.	0.4771213	13.	1.6093295
2.	2.7182818	14.	0.6213768
3.	3.1415927	15.	3.785
4.	0.7853982	16.	0.264
5.	0.5235988	17.	0.8450980
6.	0.4342945	18.	15.4323487
7.	0.2908882	19.	1.4142136
8.	4.8104774	20.	1.7320508
9.	2.5399772	21.	2.2360680
10.	0.3937043	22.	0.3819660
11.	0.3047973	23.	0.6180340
12.	3.2808693	24.	0.3010300

52. Add together numbers 2, 3, 4, from the last article; numbers 6, 8, 9; numbers 12, 9, 8; numbers 13, 18, 15; numbers 11, 24, and 14; numbers 10, 22, and 6.

53. Add the numbers marked 24, 23, 22; 23, 21, 20; 22, 20, 19; 21, 19, 18; 18, 17, 16; 15, 14, 13; 14, 12, 11; 13, 12, 10; 11, 10, 9; 10, 12, 8; 11, 6, 5; 2, 3, 5.

54. Add the numbers marked 2, 3, 4, 5, 6; 3, 5, 8, 9, 10; 4, 6, 8, 10, 11; 10, 12, 13, 6, 8; 13, 11, 10, 5, 6; 13, 14, 15, 16, 18; 14, 16, 18, 10, 12; 15, 16, 18, 19, 20; 18, 19, 21, 22, 23.

55. Add the numbers marked 1, 7, 13, 17, 22, 23, 24; 2, 3, 4, 6, 8, 9, 10; 24, 23, 22, 21, 20, 19, 18; 13, 14, 15, 16, 18, 19, 20; 10, 11, 12, 13, 14, 15, 16; 15, 18, 14, 19, 12, 11, 8.

56. A simple test of the correctness of an addition is to add a second time, beginning at the top instead of the bottom of the columns, or to add two columns at once.

It is of great advantage to educate the eye to take in at a glance digits enough to make 10 or more, and then these *sums* can be added instead of the separate digits.

$$\begin{array}{r} 61803 \\ 43429 \\ 47712 \\ 62138 \\ \hline 215082 \end{array}$$

To illustrate, take the example in the margin. Adding from the bottom, the computer says to himself (seeing $8+2=10$, and $9+3=12$), 10, 12, **22**; **8**; 8, 12, **20**; 11, 4, **15**; 11, 10, **21.**

In the two-column mode he says, 50, 32, **82**; and writes down the 82. Then he continues, 98, 52, **150**, and writes the 50 at the left of 82; then he proceeds, 11, 10, **21**, and writes the 21.

Many bookkeepers and merchants strongly recommend the addition of two columns at once, as the most expeditious and the least liable to error.

Many computers begin at the bottom of the right-hand column in adding, and write on a piece of waste-paper the full sum of each column or double column; then they begin at the top of the left-hand column, and add each column or double column, also writing the full sum; finally, they add the sums obtained in the first addition, and the sums obtained in the second addition, and compare the results. Thus, in the example above,

By Single Columns:

$$\begin{array}{r|r} 22 & 20 \\ 6 & 13 \\ 20 & 20 \\ 13 & 6 \\ 20 & 22 \\ \hline 215082 & 215082 \end{array}$$

By Double Columns:

$$\begin{array}{r|r} 82 & 213 \\ 150 & 206 \\ 20 & 22 \\ \hline 215082 & 215082 \end{array}$$

57. Add the following by double columns, and test by adding with single columns:

45.68	154.31	73.86
73.91	296.85	453.71
78.54	736.48	137.64
534.69	345.19	98.87
134.70	782.34	643.48
581.43	78.43	462.71

498.50	65.42	621.65
17.37	638.34	167.32
684.29	763.43	856.96
231.56	809.31	718.83
210.10	798.83	501.49
671.54	835.78	315.72
643.53	356.47	768.44

791.52	32.54	763.89
504.83	254.63	78.23
879.26	63.27	345.61
243.97	131.56	26.73
732.86	506.72	489.56
47.95	283.54	812.35
856.43	345.83	607.28
497.65	643.46	219.07
541.26	708.91	68.72
616.72	463.73	216.78
857.94	67.74	436.74

CHAPTER IV.

SUBTRACTION.

58. SUBTRACTION means taking away. The sign – is read **minus**, and means that the number before which it is placed is to be subtracted.

For example, $8-5=3$ is read, 8 minus 5 equals 3; $17-8=9$.

59. The number to be subtracted is called the **subtrahend**; the number from which it is taken, the **minuend**; the resulting number is called the **difference** or **remainder.**

60. Count 50 backward. Name the even numbers from 50 down to 0. Name the odd numbers from 51 down to 1.

61. Subtract by threes: beginning 60, 57; beginning 61, 58; beginning 59, 56.

62. Subtract 4 from every number between 8 and 14; between 38 and 44. Subtract 5 from every number between 5 and 15; between 85 and 95.

63. Subtract 6 from each number between 6 and 16; between 46 and 56. Subtract 7 from each number between 17 and 27.

64. Subtract 8 from each number between 18 and 28; 9 from each number between 59 and 69.

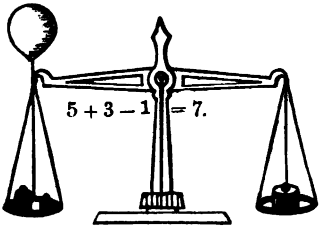
65. A number with the sign minus before it is called a **minus** number.

When the subtrahend is larger than the minuend, the remainder obtained by subtracting the minuend from the subtrahend is a minus number. Such results are common in Algebra, but are avoided in Arithmetic.

The meaning of a minus number is generally manifest; as in the example, The thermometer was at 17° and fell 25°, how high was it then? This would be written, $17^\circ - 25^\circ = -8^\circ$, and shows that the mercury fell to 8° *below* zero.

66. An expression containing the sign of equality (=) is called an **equation.** It is like a balanced scale-beam: the plus numbers may be represented by weights, the minus numbers by balloons lifting up the beam of the scale.

The one rule in working with equations is, *Keep the balance true.* In other words, *Do to one side whatever you do to the other.*



67. When numbers are connected by the signs + or −, part of them are also sometimes joined by parentheses, brackets, or bars; as,

$$7+(5-8)=4;\quad 7-(5-8)=10;\quad 7-(8-5)=4.$$
$$7+\overline{5-8}=4;\quad 7-\overline{5-8}=10;\quad 7-\overline{8-5}=4.$$

The operations on the joined numbers must be performed first, and the result treated as a single number, plus or minus.

The sign outside the parenthesis must also be prefixed; and thus arise four cases of a double sign: $++$, $+-$, $-+$, and $--$.

The $++$ is equivalent to a single $+$; the $+-$ and the $-+$ are each equivalent to a single $-$; while the $--$ means the *taking away of a minus;* it is equivalent to the *removal of a balloon*, and this is the same as *adding the weight* which the balloon was lifting; that is, $--$ is equivalent to $+$.

68. The parenthesis, therefore, may be, according to the signs, useless or needful; it may or may not affect the result, and must be handled carefully. To illustrate:

$$8-3+5=(8-3)+5=10;\ \text{but}\ 8-(3+5)=0.$$
$$12-(6-3)=12-3=9.$$
$$12-(3-6)=12-(-3)=15.$$

69. Write the second members to the following equations:

$8-3-2=$ $\quad(8-3)-2=$ $\quad 8-(3-2)=$
$8-2-3=$ $\quad(8-2)-3=$ $\quad 8-(2-3)=$
$18-(7-3)=$ $\quad 18-(3-7)=$ $\quad 18-3+7=$

70. The following questions will illustrate the meaning of minus numbers:

Starting 90 miles south of Chicago, I go 50 miles due north; and the next day, 80 miles still north. How far from Chicago am I now?

With only 67 dollars I undertake to pay three bills, of $47, of $13, and of $11. Can I pay the bills? How much shall I lack?

71. *If we add the same number to a minuend and to a subtrahend, we do not alter the remainder.* For example, $27-15=12$; and if we add any number whatever, both to 27 and to 15, the difference of the sums will be twelve; thus, adding 7 to 27 and 15, we have $34-22=12$; adding 2 to 27 and 15, we have $29-17=12$, etc.

To add 10 to any digit in a number is the same as adding 1 to the next left-hand digit. This will explain the last clause in the following rule for subtraction:

Write the subtrahend under the minuend, point under point, and each order under its corresponding order. If the number of places in the two be unequal, mentally consider the missing places filled with naughts.

Begin at the right, and subtract each figure of the subtrahend from the figure over it, setting the remainder below. But,

If the minuend figure be the smaller, add ten to it mentally; and, after subtracting, add 1 mentally to the next left-hand place in the subtrahend.

For example, in the margin we have $7-0=7$; $11-8=3$; $2-2=0$; $6-5=1$. The addition of 10 to the 1 hundredth in the minuend is exactly balanced by the subsequent addition of 1 to the 1 tenth in the subtrahend. In precisely the same manner, the following subtraction is performed, by saying, mentally, 1 from 10, 3 from 8, 7 from 11, 5 from 10, 2 from 10, 9 from 11, 1 from 7; each time setting down the remainder.*

$$\begin{array}{r@{.}l} 6&217\\ 5&18\\ \hline 1&037 \end{array}$$

$$\begin{array}{r@{.}l} 71&0018\\ 8&14721\\ \hline 62&85459 \end{array}$$

72. It is plain, that *the remainder added to the subtrahend equals the minuend; and that the remainder subtracted from the minuend leaves the subtrahend.*

These are two tests of accuracy, or checks on subtraction.

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* This addition of ten to both minuend and subtrahend is, for practical purposes, far superior to the method of transposing values in the minuend, and is as readily explained.

73. Subtract 123 from each of the numbers:
234, 343, 424, 555, 676, 725, 839, 999, 1000, 10101, 5120.

74. Subtract 456 from each of the numbers:
789, 879, 978, 6378, 6855, 6853, 7797, 7006, 3542, 4334, 9790, 3455.

75. What is the difference between 779 and 974? 368 and 249? 479 and 2301? 2731 and 929? 708 and 394? 1123 and 1072? 891 and 773? 5621 and 8103? 19,001 and 3456? 792 and 2180?

76. Subtract:
$76.47 from $183.45; $628.74 from $716.43; $549.64 from $647.51; $128.31 from $270.04; $101.50 from $125; $129.47 from $247.93; $333.95 from $641.87; $29.89 from $56.27.

77. Subtract from 7854 each of the numbers:
788, 879, 567, 5006, 6107, 578, 867, 894, 463, 4603.

78. Subtract:
2.7182818 from 3.1415927; 0.5235988 from 0.7853982; 0.4342945 from 4.8104774; 0.3937043 from 2.5399772; 0.3047973 from 0.3937043; 0.3047973 from 3.2808693; 1.6093295 from 3.2808693; 0.6213768 from 3.785; 0.264 from 15.4323487; 1.4142136 from 1.7320508; 0.3819660 from 2.2360680; 0.3010300 from 0.6180340; 1.7320508 from 2.2360680; 0.6180340 from 2.2360680; 0.3010300 from 0.3819660.

79. Subtract 0.7853982 from 3.1415927; from 2.3561945; from 1.5707963.

Subtract 0.5235988 from 3.1415927; from 2.6179939; from 2.0943951; from 1.5707963; from 1.0471975.

80. Subtract 0.3819660 from 1:
0.6180340 from 1.4142136. Add 0.3819660 to 0.6180340.

81. In a school of 83 pupils, 37 are girls; the rest, boys. How many boys are there?

82. Take 1787 from 21,205, and what is the remainder?

83. Into a bowl containing 338 fine shot I poured a handful more, and the bowl then contained 720. How many did I pour in?

84. From a box containing 209 oranges I took a basketful, and left 163 oranges. How many did I take in the basket?

85. The minuend being 1718.754, and the subtrahend 1389.328, what is the remainder?

86. If the minuend was 6532.18, and the remainder 1916.47, what was the subtrahend?

87. How many must be taken from 729,434 in order to leave 613,488?

88. How many must be taken from 1,000,000 to leave 817,259?

89. Subtract 4187.94 from 8010.101.

90. Find the difference between 8,765,420 and 9,873,210.

REVIEW II.

Addition is the process of combining two or more numbers so as to form a single number. The result obtained by adding two or more numbers is called their **sum.**

For convenience, numbers to be added are written so that figures expressing the same order shall fall in the same column.

In adding numbers, first add all the units of the lowest order; then all the units of the next higher order, and so on, reserving from the sum of the numbers of any order the *tens* of that order, to be added as *ones* to the next higher order.

Only **units of the same kind** can be added; so many things of one kind added to so many things of the *same* kind.

To test the accuracy of the work, add in the inverse order, or by double columns.

The sign + is read "plus," and is used to indicate addition. The sign = is read "is equal to," and is placed between expressions which are equal.

Subtraction is the process of finding the difference between two numbers, or of finding the number that must be added to one of the two numbers in order that their sum shall be equal to the other.

The **subtrahend** is the number to be subtracted; the **minuend** is the number from which the subtrahend is to be taken; the **difference** is the number sought.

The addition of the same number to both the subtrahend and minuend does not affect the difference. Hence, we may add ten to any order of the minuend if we add one to the next higher order of the subtrahend.

For convenience, the figures of the subtrahend are written under those of the minuend so that those of the same order shall fall in the same column. If a decimal extend to the right more places in one than in the other, the missing places are supplied mentally with zeros.

First subtract the number of the lowest order in the subtrahend from that of the same order in the minuend. If the number in any order in the subtrahend be greater than the number in the corresponding order in the minuend, add *ten* to that number in the minuend before subtracting, and then add *one* to the number of the next higher order in the subtrahend.

The sign — is read "minus," and is used to indicate subtraction.

Only **units of the same kind** can be subtracted; so many units of one kind subtracted from so many units of the *same* kind.

To test the accuracy of the work, add the subtrahend and difference together; and if the work be correct, their sum will be equal to the minuend.

Whenever a question arises in which the minuend is smaller than the subtrahend, we may subtract the minuend from the subtrahend, and prefix the minus sign to the result. Such a result is called a minus number.

CHAPTER V.

APPLICATIONS OF ADDITION AND SUBTRACTION.

91. In a till are $391 in bills, $67.50 in gold, $39.75 in silver, and $2.77 in copper and nickel. How much money is in the till?

92. Starting out with $315.75 in one wallet and $54.37 in another, I pay the grocer $127.38; the butcher, $64.17; the shoemaker, $21.40; the landlord, $50; the tailor, $35. What ought I to have left?

93. On a bill of $753.43, I pay $517.87. How much do I still owe? If I owe $817.87, and have but $637.50, how much do I lack of being able to pay?

94. If a man was born January 1, 1812, how old was he January 1, 1878? How old December 31, 1857?

95. America was discovered in 1492. How many years after its discovery was each of the following events?

Settlement of Florida, 1565; of Virginia, 1607; of Massachusetts, 1620; of Quebec, 1608; French and Indian War, 1756; Declaration of Independence, 1776; inauguration of Washington, 1789; war with England, 1812; Mexican War, 1846; Civil War, 1861.

96. How many days in common years, and in leap-years, between January 1 and March 1? January 4 and April 4?

February 5 and May 5? February 7 and October 7? January 4 and July 4? March 4 and July 4?

97. The sum of two numbers is 3; their difference, 1. What are the numbers? The sum of two numbers is 5; their difference, 1. Required the numbers. What two numbers added together make 8, if the difference of the numbers is 2? If the difference is 0? if 4? if 6?

98. If the minuend is 9874, and remainder 3185, what is the subtrahend? The subtrahend being 7659, and remainder 675.68, what is the minuend?

99. The smaller of two numbers is 7.95764328; their difference is .00087692. What is the larger number?

100. The larger of two numbers is 7.95764328, and their difference is 7.153485. What is the smaller number?

101. A hired man pumps out of my cistern in one hour 243.75 gallons; in the next hour, 227.5 gallons; in 45 minutes more, an additional 137.75 gallons; and the cistern is empty. How much was in it?

102. From what number must I subtract 5 to leave 7? 8 to leave 9? From what number must I subtract 5.1736 to leave 8.1964? 6.231 to leave 9.6648? 74.213 to leave 25.787?

103. What must be subtracted from 1 to leave .5? to leave .53? to leave .532? to leave .5236? to leave .5235988?

104. I start on a journey of 3433 miles. The first day I make 428 miles; the second day, 511 miles; the third, 497 miles; the fourth, 513. How many miles of my journey remained for me at the close of each day? How many miles had I gone at the close of each day?

105. Subtract 76,343 from the sum of 61,932, 51,387, 5193, 4674, and 8199; then subtract 23,657 from the remainder.

106. J. bought a farm and stock for $7633.90; sold off the stock for $305.75; then sold the farm for $7325. What did he lose?

107. If I gave $4375 for my land, and paid for house, barn, sheds, and fences, $2789.50; also $973.75 for horses, cattle, tools, etc.; what did my farm and stock cost?

If I sold part of the land for $675, and some cattle, etc., for $217.50, what may I estimate as the cost of what I have left?

108. Alfred the Great died at the age of 52, A.D. 901. In what year was he born? William the Conqueror began to reign A.D. 1066, and reigned 21 years. In what year did he die? Socrates was born B.C. 469, and died at the age of 70. In what year did he die? Plato was born B.C. 429, and died at the age of 82. In what year did he die? Demosthenes died at the age of 60, B.C. 322. In what year was he born? The battle of Marathon was fought B.C. 490; 560 years later Jerusalem was destroyed by Titus. In what year was Jerusalem destroyed?

109. John has 158 cents, James has 271 cents; James gives John 56 cents. Which has more than the other, and how many more?

CHAPTER VI.

MULTIPLICATION.

110. A St. Andrew's cross × between two numbers means that one of the numbers is to be repeated as many times as is indicated by the other number. The number to be repeated is called the **multiplicand**; the number which shows how many times the multiplicand is to be repeated is called the **multiplier**; and the result is called the **product.**

The sign × is read **times,** or **multiplied by,** according as the multiplier precedes or follows the multiplicand. Thus, 5×4 cents $= 20$ cents is read, five times four cents equals twenty cents; but, 4 cents $\times 5 = 20$ cents is read: four cents multiplied by five equals twenty cents.

111. The multiplier and multiplicand are often called **factors** of the product. The product of two or more factors is the same in whatever order they are taken. Thus, $3 \times 4 = 4 \times 3$. The dots in the margin, read horizontally, make 3 fours; read vertically, make 4 threes.

● ● ● ●
● ● ● ●
● ● ● ●

112. The sign × cannot extend its power, forward or backward, beyond a + or –, without the aid of a parenthesis. To illustrate:

$$2+3\times4-1=13; \qquad 2+3\times(4-1)=11;$$
$$(2+3)\times4-1=19; \qquad (2+3)\times(4-1)=15.$$

113. The products, in all cases in which neither factor exceeds ten, should be thoroughly committed to memory. They will be found in the following table:

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108

114. In the above table, take the multiplier in the upper line, the multiplicand in the left-hand column; the products will be found directly under the multiplier, and opposite the multiplicand; as, 12 × 7 is 84.*

115. To multiply any multiplicand by a multiplier less than 13, the work may be written as in the margin. Beginning at the right, 4 × 8 is **32**; the 2 is written, and the 3 carried mentally and added to 4 × 6, making **27**; and the process is thus continued to the left.

$$\begin{array}{r} 2.236068 \\ 4 \\ \hline 8.944272 \end{array}$$

$12 \times 8 = \mathbf{96}$; $12 \times 6 = 72$, and 9 makes **81**; $12 \times 6 = \mathbf{72}$; $12 \times 3 = 36$, and 7 makes **43**; $12 \times 2 = 24$, and 4 makes **28**; $12 \times 2 = 24$, and 2 makes **26**.

$$\begin{array}{r} 2.236068 \\ 12 \\ \hline 26.832816 \end{array}$$

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* The table should be learned, not by lines but by squares; that is, first learn 2 × 2, 2 × 3 and 3 × 2, 3 × 3; next learn 2 × 4, 3 × 4, 4 × 2, 4 × 3, 4 × 4; thirdly, 2 × 5, 3 × 5, 4 × 5, 5 × 2, 5 × 3, 5 × 4, 5 × 5; fourthly, all products under 36, etc.

The cards referred to in the footnote to § 43 may be advantageously used for practice in multiplying two digits. Shuffle them, and pass them in couples from one hand to the other, naming the two factors, while the pupil names the products.

116. Multiply 111 by 5; 123 by 3; 231 by 2; 114 by 3; 421 by 4; 512 by 5; 4328 by 4; 1187 by 6; 1782 by 8; 8.287 by 7; 9.6198 by 3; 62.818 by 7; 9.2758 by 8; 52.134 by 9.

117. Multiply .5235988 by 6; .7853982 by 4; 3.14159265 by 5, and the product by 5.

118. Multiply 3.1416 by 11; by 12; by 10 and by 3, and add the two results; by 10 and by 4, and add the results; by 9 and by 6, and add the results. Multiply 2.236068 by 11; by 6 and by 7, and add the results; by 8 and by 9, and add the results; by 10 and by 7, and add the results (compare the sum of these two products with the sum of the last two products); by 10 and by 8, and add the results; by 12 and by 7, and add the results.

119. To multiply by 10, 100, 1000, etc., it is enough to move the decimal point of the multiplicand* as many places to the *right* as there are *naughts* after the one; annexing naughts to the multiplicand, if necessary.

Thus, 10 × 15.4323 = 154.323; 10,000 × 15.43 = 154,300.

120. How much is 10 times 3.14159265? 100 times? a million times? What will 10 barrels of apples cost, at $3.75 a barrel? at $2.17? at $5.875? How much will 100 barrels cost at each of these prices, and at $3.375? at $5.125?

121. It is plain, that to multiply by .1, .01, .001, etc., it is necessary only to move the point of the multiplicand as many places to the *left* as there are *decimal places* in the multiplier, prefixing naughts to the multiplicand, if necessary.

---

* When the multiplicand is a whole number, the decimal point is not written, but understood, after the units' figure.

Thus, $.1 \times 15.43 = 1.543$; $.001 \times 15.43 = .01543$.

122. What is a tenth of 2.36? a hundredth of 2.36? a thousandth of .63? Write the second members of the following equations, and then read them:

$.01 \times 7.8 =$ $\quad .001 \times 4.31 =$ $\quad .0001 \times 23.31 =$

$.1 \times .065 =$ $\quad .01 \times .012 =$

123. It is evident that 30 times is 3 times as much as 10 times, or 10 times as much as 3 times. Hence (§§ 115, 119),

Find the cost of 30 barrels of flour, at $3.27 a barrel; of 70 barrels, at $4.58; of 90 barrels, at $6.76; of 100 barrels, at $7.84; of 120 barrels, at $8.57.

124. It is evident that $.03 = 3 \times .01$. Hence (§ 121),

Find the cost of .03 of a barrel of oil, at $27.875 a barrel; of .7; of .009; of .17*; of .019; of .13; of .8; of .83; of .014 of a barrel.

It is plain from the above examples that,

The decimal places in the product are as many as the sum of the decimal places in the multiplicand and the multiplier.

125. What is the numerical value of the expressions:

30×8.75? $\quad 700 \times 7.81$? $\quad 300 \times 7.85$?

$.07 \times 6.975$? $\quad 8000 \times 65.432$? $\quad .0009 \times 10356.78$?

126. Ex. 1. Multiply 6957 by 463. $463 = 400 + 60 + 3$.

If we wished to find the result by addition, it would be necessary to write 463 numbers, each equal to 6957, under each other, and find the sum of all these numbers. (The necessity of abridging such an operation has given rise to multiplication.)

---

* Multiply first by .1, then by .07, and add the results.

Or, we might make three additions, the first of 3 numbers, each equal to 6957; the second of 60 numbers, each equal to 6957; the third of 400 numbers, each equal to 6957; and then add together the three sums. In other words, we might multiply 6957, first by 3, then by 60, and then by 400, and find the sum of the three products obtained. Thus,

$$\begin{array}{rrl}
\text{Multiplicand,} & 6957 & \\
\text{Multiplier,} & \underline{463} & \\
\text{3 times the multiplicand} = & 20871 & \multirow{3}{*}{$\left.\vphantom{\begin{array}{c}0\\0\\0\end{array}}\right\}$ Partial Products.} \\
\text{60 times the multiplicand} = & 417420 & \\
\text{400 times the multiplicand} = & 2782800 & \\
\hline
\text{463 times the multiplicand} = & 3221091 &
\end{array}$$

It is evident that the zeros at the right of the second and third partial products do not affect the result of the addition; we may, then, omit them, if we observe *to what place in respect to the next preceding product* the right-hand figure of each succeeding product belongs.

Therefore, *multiply by each digit of the multiplier, place the right-hand figure of each product under the multiplier used, and add the partial products.*

127. Ex. 2. Multiply 1231 by 2007.

Solution of Ex. 2.

$$\begin{array}{r}
1231 \\
\underline{2007} \\
8617 \\
2462 \\
\hline
2470617
\end{array}$$

Ex. 3. Multiply 964.73 by 123.8.

Solution of Ex. 3.

$$\begin{array}{r}
964.73 \\
1238 \\
\hline
771\,784 \\
2894\,19 \\
19294\,6 \\
96473 \\
\hline
119433.574
\end{array}$$

In multiplication, write the units' digit of the multiplier under the right-hand digit of the multiplicand. Put the right-hand digit of each partial product under the multiplying digit used; and put the decimal point in the product directly under the decimal point in the multiplicand.

The importance of putting each partial product in its proper place is seen from Ex. 2, and the method of dealing with the decimal point from Ex. 3.

128. A test of the accuracy of the pointing is furnished, by observing that (by § 124),

The fractional places in the product are as many as the sum of the fractional places in the factors.

Care must be taken lest the omission of superfluous naughts confuse us. Thus, $6000 \times .5236 = 3141.6$, which appears like a violation of the rule until we notice that three 0's are omitted in the product.

Another test is furnished by making a rough estimate of what the product should be. Thus, in the last example, we have 6000 times a number a little greater than .5; the product, therefore, should be something over 3000.

129. Multiply .785398 by each of the following numbers: 2; 20; 3; 300; 5; .5; .005; 737; 7.37; 856; 85.6; .0856; 10; 1001; 1.001; 954; .00954.

130. Multiply 2150.42 by .1; by .001; by .75; by .075; by .083.

131. Multiply 1.4142136 by .7; by .707; by .7071; by .707107. Multiply 1.41421 by 1.4; by 1.4142; by 1.41422. Multiply 1.732 by 1.732; 2.23607 by 2.236; .618 by 618; .618034 by .618035. Subtract this last product from 1.

132. When the multiplier is the product of factors, it is occasionally a saving of labor to multiply by the factors successively; as, for example, to multiply by 289, we may multiply by 17 and then multiply the product by 17; or, to multiply by 8.4, we may multiply by .7 and by 12.

133. Find the value of the expressions:

88 × 718.54; 96 × 6.8193; 6.3 × 71.569; 1.32 × 234.769.

134. Multiply 291.47 by 16, and the product by 625. In like manner, find the continued products:

8 × 125 × 278.56; 8 × 3.75 × 3.33333; 8 × 625 × 1.5708.

135. One mile measures 5280 feet. How many feet in 3 tenths of a mile? in .7? in .17? in .573? in .846 of a mile?

136. When a number is subtracted from 1 standing in the *place next to the left of the highest figure in the number*, the remainder may be called a **complement** of the number; but if the 1 is put in the unit's place, the remainder is called **the arithmetical complement.**

For example, *the arithmetical* complement of .097 is .903; but .003 may also be called *a* complement of .097.

137. When a complement of a multiplier is a simpler factor than the multiplier, we may multiply by that complement, and subtract the product from the product obtained by multiplying by 1 in the proper *order* of units.

For example, .097 of a multiplicand is found by subtracting .003 of the multiplicand from .1 of the multiplicand; and to multiply by 9.98, we need but to subtract .02 of the multiplicand from 10 times the multiplicand.

138. Multiply (using complements) .7854 by 9.9; by .99; by .099. Multiply .5236 by 99.7; by 9.989; by 9.87. Multiply 8537 by .0097; by .9995.

139. Multiply .61803 by 147; by 373; by 7.56; by 8.93; by 9.93. Multiply .5236 by 5.99; by 7.99; by 8.997; by 699.98.

140. Multiply .7854 by .618; by .382; by .7854; by .302. Multiply 2.718 by .618; by .382; by .7854; by .607.

141. Find the continued products:
.477 × 101 × .708; 15.43 × .4343 × 3; 4 × .175 × 3.28;
.615 × .771 × 10; 3.2809 × 5 × .71; .785 × .7 × .202;
.471 × .807 × 22; 3.28 × 25 × .909.

142. In ordinary calculations we seldom use a fraction smaller than .00001 of the unit. Therefore, we generally need not retain more than five places of a decimal fraction; and if a product contains more than seven places of a decimal, we curtail it by omitting the right-hand places.

143. Whenever absolute accuracy of a long decimal is not required, in curtailing the decimal 1 must be added to the figure in the lowest place retained, if the figure in the next place is 5, or greater.

Thus, 2.2360679775 must be represented by 2.236067978, 2.23606798, 2.2360680, 2.236068, 2.23607, 2.2361, 2.236, 2.24, or 2.2, according to the degree of accuracy required.

Give the nearest values to 1.732050808 for each place of decimals from the first to the eighth.

144. When the multiplier contains many more places of significant* digits than the multiplicand, we may reverse the factors and use the multiplicand as multiplier, provided we carefully notice what we are doing, and interpret the result correctly.

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* The digits 1, 2, 3, 4, 5, 6, 7, 8, 9, are called significant digits.

1578 sheep, at $3 a head, are worth $4734; and, although in numbers we may obtain the result more easily by saying $3 \times 1578 = 4734$, yet we must remember that the $4734 is really 1578 × $3.

145. What will a man earn in a year if he has $2 a day, omitting Sundays? Suppose that the year begins on Sunday? Suppose the year to be leap-year, and not begin on Sunday? Suppose it leap-year, and to begin on Saturday?

2. If a field of corn averages 2 ears to a stalk, how many ears on 673 stalks?

3. At 27 bushels an acre, how much wheat to the square mile of 640 acres, deducting 47 acres for roads and waste land?

4. How much money would be required to give $7000 to each of 7568 men?

5. In a certain book of 378 pages, the words average 7 letters to a word, and 10 words to a line. There are, on an average, 29 lines to a page. How many letters in the book?

6. How many bushels of wheat in a township of 37 square miles, if we deduct 47 acres to the square mile for roads and waste, and suppose that half the remainder is in wheat averaging 23 bushels to an acre?

7. If 5700 persons, each paying 1 cent toll, and 324 carriages, each paying 5 cents toll, pass over a bridge in a day, how much money will be received?

8. A merchant bought 960 pounds of cheese at 7 cents a pound, and 147 pounds of butter at 20 cents. He gave in payment 12.5 yards of cloth at 1 dollar a yard, 2 barrels of sugar, each weighing 226 pounds, at 9 cents a pound, and the remainder in cash. How much money had he to pay?

In each question of this article, distinguish between the real multiplicand and number used as multiplicand.

146. If a product consist of **equal** factors, it is called a **power** of that factor, and one of the equal factors is called a **root** of the product.

The power is named according to the *number* of equal factors taken. Thus,

$5 \times 5 = 25$; 25 is the *second power*, or **square**, of 5.
$5 \times 5 \times 5 = 125$; 125 is the *third power*, or **cube**, of 5.
$5 \times 5 \times 5 \times 5 = 625$; 625 is the *fourth power* of 5.

Likewise,

5 is called the *second root*, or **square root**, of 25.
5 is called the *third root*, or **cube root**, of 125.
5 is called the *fourth root* of 625.

147. To avoid the necessity of writing long rows of equal factors, a small figure, called the **exponent**, is written at the right of a number to show how many times the number is taken as a factor. Thus,

5^5 means the same as $5 \times 5 \times 5 \times 5 \times 5$, and is read the fifth power of 5.

3^3 times 3^4 means $3 \times 3 \times 3$ times $3 \times 3 \times 3 \times 3 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7$.

148. From this last example, it is evident that,

The product of two or more powers of the same number is expressed by writing the number with an exponent equal to the sum of the exponents of the given powers.

Express the product of,

1. $7^5 \times 7^3$; $8^2 \times 8$; $2^8 \times 2$; $5^4 \times 5^2$.
2. $3.01^2 \times 3.01$; $0.67^2 \times 0.67^8$; $.208 \times .208^3$.
3. $2.003^2 \times 2.003^4$; $20.03^3 \times 20.03$; 20.03×20.03^2.

REVIEW III.

Multiplication, strictly speaking, is the **operation** of finding the **sum** of two or more **equal** numbers.

With reference to this operation, this sum is called the **product**; one of the equal numbers is called the **multiplicand**; and the number which shows *how many times* the multiplicand is taken is called the **multiplier.**

In multiplication *we must always have* **two different kinds of units**: *So many* **things** *taken or repeated so many* **times.**

It is sometimes required to take or repeat the **whole** of the multiplicand, sometimes a **certain part** of it, sometimes the **whole and a certain part** of it; and the meaning of multiplication is *extended* to cover all of these cases.

To multiply by a **whole number** is to take the multiplicand as many times as is indicated by the whole number.

To multiply by a **fraction** is to take **such a part of** the multiplicand as is indicated by the fraction.

To multiply by a **whole number and a fraction** is to take the multiplicand as many times as is indicated by the whole number, and such a part of the multiplicand as is indicated by the fraction.

The product, therefore, will be equal to, greater than, or less than the multiplicand, according as the multiplier is equal to, greater than, or less than one.

The multiplicand and the multiplier are often called **factors** of the product.

When a product consists of more than two factors, it is called the **continued product** of the factors.

When a product consists of two or more **equal** factors, it is called a **power** of that factor; and one of the equal factors is called a **root** of the product.

The **index** or **exponent** of a power is a small figure placed at the right of a number to show how many times the number is taken as a factor, and is read the *first* power, *second* power, *third* power, etc., of the number.

The second power of a number is generally called the **square** of the number, and the third power is called the **cube** of the number.

The product of two or more powers of the same number may be expressed by writing the number with an exponent equal to the **sum of the exponents** of the given powers.

The numerical result of multiplying one number by another is the same whichever is taken as the multiplicand, the other being taken as the multiplier; *but the product will always denote the same kind of units as the true multiplicand.*

A St. Andrew's cross × placed between two numbers means that one of the numbers is to be multiplied by the other, and it is read "times," or "multiplied by."

To multiply by 10, 100, 1000, etc., it is necessary only to move the decimal point in the multiplicand as many places to the *right*, annexing ciphers, if necessary, as there are *ciphers* in the multiplier.

To multiply by .1, .01, .001, it is necessary only to move the decimal point in the multiplicand as many places to the *left*, prefixing ciphers, if necessary, as there are *places* in the multiplier.

The decimal places of a product are equal in number to the decimal places in the multiplicand and multiplier counted together.

To multiply by the product of two or more factors gives the same result as to multiply by one of the given factors, this product by another of the factors, and so on.

If we have to multiply one number by another, **we may separate the multiplicand into parts,** multiply each part by the multiplier, and add the results to form the complete product. Or, **we may separate the multiplier into parts,** find the product of the multiplicand by each part, and add these partial products to form the complete product.

When the **multiplier is a single digit,** the product is obtained by multiplying each *order* of the multiplicand, beginning with the lowest order, care being taken to write down the ones of each partial product, and to reserve the *tens* to be added as *ones* to the product of the next higher order.

When the **multiplier consists of two or more digits,** multiply by each digit separately, arrange the partial products so that figures of the *same order* shall fall in the *same column*, and add.

To test the accuracy of the work, interchange multiplicand for multiplier. Or, separate the multiplier into parts, multiply the multiplicand by each part separately, and add the results. If the work be correct, their sum will be equal to the original product.

It is best to use the sign × in the sense of "times"; that is, to write the multiplier before the sign, and the multiplicand after it. The other order is allowed, but it is not so good. See § 146.

CHAPTER VII.

DIVISION.

149. Division is the operation by which, when a product and one of its factors are given, the other factor is found.

With reference to this operation, the product is called the **dividend**; the given factor is called the **divisor**; and the required factor is called the **quotient.**

150. When the given factor is the multiplicand, the factor sought is the multiplier.

In this case, the question is: *What must we* **multiply the divisor by** *to get the dividend?*

To answer this question, it is necessary to find *how many times* the divisor is contained in the dividend, and the answer will be **so many times.**

Thus, in the question, How *many times* are 6 cents contained in 30 cents? the factor sought is the multiplier, 5; 5 times 6 cents are 30 cents.

When the given factor is the multiplier, the factor sought is the multiplicand.

In this case, the question is: *What must we* **multiply by the divisor** *to get the dividend?*

To answer this question, it is necessary to divide the dividend into as many **equal parts** as is indicated by the number in the divisor, and the answer will be **so much to each part.**

Thus, in the question, How *much* will each boy receive if 30 cents be divided among 6 boys? the factor sought is the multiplicand, 5 *cents;* 6 times 5 cents are 30 cents.

The *arithmetical process* and the *numerical result* are the same in both cases; but the *name* to be attached to the result depends upon the nature of the question.

151. Division is indicated by the sign ÷, by the colon, :, or by writing the dividend over the divisor, and drawing a line between them; thus, each of the expressions, $15 \div 3$, $15:3$, $\frac{15}{3}$, means and is read, "Fifteen **divided by** three."

SHORT DIVISION.

152. When the divisor does not exceed 12, the work may be written in the following manner: the divisor is placed to the left of the dividend, the quotient under the dividend, and each remainder is added as so many *tens* to the next figure of the dividend not divided.

(1) Divide 2736 by 4.

$$\begin{array}{r} 4\overline{)2736} \\ \hline 684 \end{array}$$

Wording. 4 in 27, **6**, carry 3; in 33, **8**, carry 1; in 16, **4**. *Answer*, 684.

(2) Divide 2736 by 9.

$$\begin{array}{r} 9\overline{)2736} \\ \hline 304 \end{array}$$

Wording. 9 in 27, **3**; in 3, **0**, carry 3; in 36, **4**. *Answer*, 304.

(3) Divide 3696 by 12.

$$\begin{array}{r} 12\overline{)3696} \\ \hline 308 \end{array}$$

Wording. 12 in 36, **3**; in 9, **0**, carry 9; in 96, **8**. *Answer*, 308.

The pupil will observe that, the divisor being a whole number, each quotient figure is of the same **order of units** *as the right-hand figure of the partial dividend used in obtaining it.*

153. Divide 963 by 3; 846 by 2; 846 by 3; 846 by 6; 848 by 4; 52.05 by 5; 84.028 by 7; 13.31 by 11; 1.728 by 12.

When the divisor is a whole number, the pupil will be careful to put the decimal point in the quotient as soon as the decimal point in the dividend is reached.

154. If the divisor is not contained in the dividend without a remainder, ciphers may be mentally annexed to the dividend, and the division continued until the required number of decimal places is obtained. *If the remainder, when the last quotient figure is obtained, is greater than half the divisor, increase the quotient figure by* 1.

Divide 3.1 by 4, by 5, and by 7, carrying the division to the third place of decimals.

$$\begin{array}{r} 4\underline{)3.1\ \ } \\ .775 \end{array} \qquad \begin{array}{r} 5\underline{)3.1\ \ } \\ .620 \end{array} \qquad \begin{array}{r} 7\underline{)3.1\ \ } \\ .443 \end{array}$$

155. If the dividend and divisor are both multiplied, or both divided, by the same number, the quotient is not changed.

Thus, $12 \div 4 = 3$, and (when both dividend and divisor are multiplied by 2) $24 \div 8 = 3$. Again (when both dividend and divisor are divided by 2) $6 \div 2 = 3$.

Hence, if we have to divide 2.24 by 35, we may first divide by 7, and then by 5, as follows:

$$\begin{array}{r} 7\underline{)2.24} \\ .32 \end{array} \qquad \text{Again,}\ \begin{array}{r} 5\underline{).32\ } \\ .064 \end{array} \qquad \textit{Answer},\ .064.$$

Dividing both dividend and divisor by the same number is called **cancelling** equal factors in dividend and divisor.

Find the quotients, to five decimal places, of: $3 \div 7$; $4 \div 11$; $3.17 \div 14$; $7.85 \div 21$.

156. Since the quotient is not altered if the dividend and divisor are both multiplied, or both divided, by the same number, it follows that,

If the divisor contains decimal places, we may remove the decimal point from the divisor, provided we carry the decimal point in the dividend as many places to the right as there are decimal places in the divisor.

If the divisor is a whole number and ends in zeros, the zeros may be cut off, provided the decimal point in the dividend is carried to the left as many places as there are zeros in the divisor.

Divide 78.52 by .008, by .8, and by 8000, first making the required changes of the decimal point.

8)78520	8)785.2	8).07852
9815	98.15	.009815

157. From the last three examples it is seen that,

If the first figure of the quotient is placed under the right-hand figure of the first partial dividend, the decimal point in the quotient will be written directly under the decimal point in the dividend.

158. Divide,*

1.	.003 by .07 ;	.003 by 110 ;	110 by .003.
2.	.07 by .003 ;	110 by .07 ;	1.3 by .07.
3.	1.7 by .07 ;	.07 by 110 ;	1.3 by 110.
4.	1.7 by 110 ;	.07 by 1.2 ;	.003 by 1.2.
5.	110 by 1.2 ;	1.7 by 1.2 ;	17 by 1.2.
6.	136 by .06 ;	136 by .12 ;	136 by 1100.
7.	256 by .8 ;	2.56 by .08 ;	.0256 by .008.
8.	256 by 8000 ;	1.06 by .9 ;	1.06 by 9000.

---

* If the divisor is not contained in the dividend without a remainder, carry the quotient to the fifth decimal place.

LONG DIVISION.

159. The process of Long Division is precisely like that of Short Division, except that the work is written in full, and the quotient is written *over* the dividend.

Divide 3.1415927 by 1.73.

Suppress the decimal point in the divisor, and move the point two places to the right in the dividend.

$$
\begin{array}{rr}
 & 1.81595 \\
173) & \overline{314.15927} \\
 & \underline{173} \\
 & 1411 \\
 & \underline{1384} \\
 & 275 \\
 & \underline{173} \\
 & 1029 \\
 & \underline{865} \\
 & 1642 \\
 & \underline{1557} \\
 & 857 \\
 & \underline{692} \\
 & 165
\end{array}
$$

173 is contained in 314 once. The one is written over the 4, and, by § 156, the point is placed over the decimal point in the dividend. The process, which has been gone through with, mentally, in short division, is written out below the dividend. 173 is subtracted from 314; to the remainder, the 1 in the dividend is annexed; then 8 × 173 is subtracted; then to the remainder, 5 in the dividend is annexed, and so on. The fifth place in the quotient is 4, but written 5, because the remainder, 165, is more than half 173.

Divide 23.13685 by 7.843.

Suppress the decimal point in the divisor, and move the point three places to the right in the dividend.

$$
\begin{array}{rr}
 & 2.95 \\
7843) & \overline{23136.85} \\
 & \underline{15686} \\
 & 74508 \\
 & \underline{70587} \\
 & 39215 \\
 & \underline{39215}
\end{array}
$$

Determine each figure of the quotient carefully before writing it. The first figure of the quotient is 2; for, though 7 is contained 3 times in 23, 78 is contained only 2 times in 231. The second figure is obviously 9; the third figure is 5, for 7 is contained 5 times in 39.

Note. As a test of the accuracy of each quotient figure, observe that if the product of the divisor by the quotient figure is greater than the partial dividend, the quotient figure is too large; and if the difference between the partial dividend and the product of the divisor by the quotient figure is greater than the divisor, the quotient figure is too small.

From these examples the following rule is deduced:

Change the decimal point so as to make the divisor a whole number, and find, by trial, the first quotient figure.

Subtract from the partial dividend the product of the divisor by this quotient figure.

Consider the remainder with the next figure of the dividend annexed as a **new partial dividend,** *and proceed as before.*

160. Divide,

1. 1.6093295 by .479; by .917; by .017; by .0087.
2. 3. by 1.7; by 1.73; by 1.732; by 1.7321.
3. 1.6093295 by 5280, and the quotient by 12.
4. 2 by 1.4142; 5 by 2.236.

161. Two numbers are **reciprocals** of each other when their product is equal to 1. Thus, .5 is the reciprocal of 2; 4 is the reciprocal of .25; 3 is the reciprocal of .33333; .75 is the reciprocal of 1.33333; .8 is the reciprocal of 1.25, etc.

162. When the dividend is 1, the quotient is the reciprocal of the divisor. And if the dividend is any other number, as 5, 7, etc., the quotient will be 5, 7, etc., times the reciprocal of the divisor; or, in other words, *the reciprocal times* 5, 7, etc.; that is,

To divide a number by a divisor gives the same numerical result as to multiply it by the reciprocal of the divisor.

163. It is equally plain, that *to multiply by a factor gives the same result as to divide by the reciprocal of the factor.*

164. The processes of multiplication and division are frequently made much simpler by using reciprocals of the multiplier or of the divisor.

It is easier to use 8, .8, .08, etc., than to use .125, 1.25, 12.5, etc. If required to divide 2.71828 by 37.5, we may (if we notice that 37.5 is 3×12.5) divide 2.71828 by 3, and multiply the quotient by .08.

165. Perform the work in the following questions by the use of reciprocals:

1. $8 \times .25 =$
2. $171 \div .25 =$
3. $876 \times 1.25 =$
4. $132 \times 2.5 =$
5. $591 \div 2.5 =$
6. $756 \div .125 =$
7. $268 \times 25 =$
8. $753 \div 25 =$
9. $567 \div 625 =$
10. $1764 \times .025 =$
11. $5381 \div .025 =$
12. $7452 \div .875 =$
13. $651 \times .33333 =$
14. $456 \times 6.66667 =$
15. $1554 \times .16667 =$
16. $432 \div 1.33333 =$
17. $375 \div 16.667 =$
18. $225 \div 6.6667 =$

166. If 8375 mills be divided by 10 (the number of mills in a cent), the quotient, 837.5, is the number of cents in 8375 mills. Divide the same number by 1000, and the quotient, 8.375, is the number of dollars.

In like manner, if we divide 173 inches by 12 (there being 12 inches in a foot), the quotient, 14.417, is the number of feet in 173 inches.

And in general, *to express a given number in terms of a second number as* **unity**, *we have only to divide by the second number.*

167. Taking 7 as unity, what would be the value of 14? of 28? of 35? of 3.5? of 2.8105? of 6.31415?

2. If the side of a square is 10 inches, and its diagonal 14.14214, express the side in terms of the diagonal as unity.

3. If the diagonal of a square is one foot, what decimal of a foot must its side be?

4. If the diameter of a circle is 11.3 inches, and its circumference 35.5 inches, what is the circumference in terms of the diameter? What is the diameter in terms of the circumference?

5. What decimal fraction of 87 is 47? 53? 43.5? 29?

6. How many times 393 is 587? 7857? 131? 196.5?

7. How many 684's are there in 1368? in 1760? in 342? in 77? in 6.84? in .0785?

8. If a meter contains 39.37 inches, and a yard 36 inches, express a yard in terms of a meter.

9. If a square meter contains 1550 square inches, and a square yard 1296 square inches, express a square yard in terms of a square meter.

10. If a cubic meter contains 61,025 cubic inches, and a cubic yard 46,656 cubic inches, express a cubic yard in terms of a cubic meter.

11. If a liter (pronounced *leé-ter*) contains 61.025 cubic inches, and a dry quart 67.2 cubic inches, express a dry quart in terms of a liter.

12. If a kilogram contains 15432.35 grains, and a pound avoirdupois contains 7000 grains, express a kilogram in terms of a pound.

13. The diameter of a circle is 5.5 feet, and the circumference is 17.2788 feet. If the diameter is denoted by unity, what will denote the circumference?

14. A wheel is 36 inches in diameter, and its axle is 6.2 inches. If the diameter of the wheel is expressed by unity, what will express the diameter of the axle?

DIVISION OF POWERS.

168. Since $5^5 = 5 \times 5 \times 5 \times 5 \times 5,$

and $5^3 = 5 \times 5 \times 5,$

therefore, $\frac{5^5}{5^3} = \frac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5} = 5 \times 5 = 5^2.$

That is,

(1) *The quotient of two powers of the same number is expressed by writing the number with an exponent equal to the exponent of the dividend minus that of the divisor.*

Also, $\frac{5^3}{5^5} = \frac{5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5} = \frac{1}{5 \times 5} = \frac{1}{5^2}.$

But by (1), $\frac{5^3}{5^5} = 5^{-2};$

therefore, $5^{-2} = \frac{1}{5^2}.$ That is,

(2) *Any number with a* **minus** *exponent is the* **reciprocal** *of the number with an equal* **plus** *exponent.*

Again, $\frac{5^3}{5^3} = 1.$

But by (1), $\frac{5^3}{5^3} = 5^0;$

therefore, $5^0 = 1.$ That is,

(3) *Any number with* **zero for an exponent** *is equal to* 1.

Find the value of

1. 10^0; 10^1; 10^2; 10^3; 10^4; 10^5; 10^6.
2. 10^0; 10^{-1}; 10^{-2}; 10^{-3}; 10^{-4}; 10^{-5}; 10^{-6}.
3. $10^0 \times 10^0$; $10^1 \times 10^{-1}$; $10^2 \times 10^{-2}$; $10^3 \times 10^{-3}$; $10^4 \times 10^{-5}$.
4. $10^3 \div 10^{-1}$; $10^{-2} \div 10^2$; $10^{-1} \div 10^{-3}$; $10^{-2} \div 10^{-4}$.
5. $10^{-3} \times 10^2$; $10^3 \div 10^2$; $10^{-3} \div 10^2$; $10^{-2} \div 10^{-3}$.
6. $10^2 \div 10$; $10^2 \div 10^3$; $10^0 \div 10^{-1}$; $10^{-1} \div 10^{-1}$.
7. $1.01^2 \div 1.01^{-1}$; $1.01^2 \times 1.01^{-1}$; $1.01^{-2} \div 1.01^{-1}$.

REVIEW IV.

Division is the operation by which, when a product and one of its factors are given, the other factor is found.

With reference to this operation, the product is called the **dividend**; the *given* factor, the **divisor**; and the *required* factor, the **quotient.**

When the given factor is the multiplicand, the factor sought is the multiplier.

In this case, it is required to find *what we must* **multiply the divisor by** *to get the dividend.*

It is, therefore, necessary to find *how many times* the divisor is contained in the dividend; and the answer will be, **so many times.**

When the given factor is the multiplier, the factor sought is the multiplicand.

In this case, it is required to find *what we must* **multiply by the divisor** *to get the dividend.*

It is, therefore, necessary to divide the dividend into as many *equal parts* as is indicated by the number in the divisor; and the answer will be, **so much to each part.**

The **arithmetical process** and the **numerical result** are the same in both cases, *but the name to be attached to the quotient depends upon the nature of the question.*

Division is indicated by the sign ÷, the colon, : or by writing the dividend over the divisor and drawing a line between them. Whatever symbol of division is employed, it is read, "divided by."

The dividend may be separated into parts, and each part divided by the given divisor; the sum of the results will then be the entire quotient.

Hence, in dividing,

Find by trial the first figure of the quotient.

Subtract from the partial dividend the product of the divisor by this quotient figure.

Consider the remainder with the next figure of the dividend annexed as a **new partial dividend,** and proceed as before.

When the divisor is so small that the necessary multiplications and subtractions can be carried on mentally, the operation is called **Short Division.**

When the work is written in full, the operation is called **Long Division.**

The decimal point may be suppressed in any divisor, if the point in the dividend be moved as many places to the right as there are decimal places in the divisor. And if the first quotient figure be written over the right-hand figure of the first partial dividend, the position of the decimal point in the quotient will be directly over that in the dividend.

To divide by 10, 100, 1000, etc., it is necessary only to move the decimal point in the dividend as many places to the *left* as there are *ciphers* in the divisor.

To divide by .1, .01, .001, etc., it is necessary only to move the decimal point in the dividend as many places to the *right* as there are *decimal places* in the divisor.

To divide by the product of two or more divisors gives the same result as to divide first by one divisor, the quotient by another divisor, and so on. That is,

Equal factors of the dividend and divisor *may be suppressed without altering the value of the quotient.*

Suppressing equal factors in dividend and divisor is called **cancelling.**

When the dividend is 1, the divisor and quotient are **reciprocals** of each other. Hence,

Multiplying by the reciprocal of a number is equivalent to dividing by the number.

Dividing by the reciprocal of a number is equivalent to multiplying by the number.

The quotient of a power by another power of the same number may be expressed by writing the number with an exponent equal to the exponent of the dividend diminished by the exponent of the divisor.

Any number with a **minus** exponent is the **reciprocal** of the number with an equal **plus** exponent.

The quotient of two equal powers of the same number may be expressed by unity, or by the number with zero for an exponent. That is,

The **zeroth** power of any number is equal to 1.

To test the accuracy of the work in Division, multiply the divisor by the quotient. The product should be equal to the dividend.

When a parenthesis includes two or more numbers, the included numbers must first be reduced to a single number, and the result put in place of the parenthesis.

CHAPTER VIII.

EXERCISE I.

Express in words:

1. 327.244.
2. 80.9056.
3. .390012.
4. 20000.002.
5. .0000008.
6. 41.27105.

Write in figures:

7. Two hundred thirty-five and eight hundred thirty-five thousandths.

8. Seventy-four and two hundred three thousand six millionths.

9. Twelve hundred and eight thousand three ten-millionths.

10. Five thousand sixty-four millionths.

11. One million and four tenths.

12. Six hundred-millionths.

Multiply and divide:

13. 789.365 by 10; by 100; by 100,000.

14. .004 by 100; by 10,000; by 1000.

15. 436. by 1,000,000; by 1000; by 10.

16. .1 by ten; by ten millions.

Find the value of:

17. $21.3706 + 15.243 + 1.8954 + .026891 + 5.328 + 29.74$.

18. $57. + .0057 + 6.8 + 1200 + .847 + 159.2 + 3$.

19. $.0012 + 10 + 5.8281 + 5 + 39.43 + .6827 + 1$.

20. 23.9875 − 12.4764; 35.14732 − 27.62815.

21. 102.1274 − 83.072; 39.801 − 17.9645.

22. 30 − 5.2817; 1.7 − .8469.

23. 1 − .54237; 100 − .00176.

24. 24.271 − 3.6485 + 15.271 − 13.256 − 14.125.

25. 52 + .52 − 17.8946 − 30.254 − .5 + 21.12.

26. 41.289 × .5; .268 × .9; .112 × .2.

27. 2.435 × 4.23; 71.651 × 3.37; .251 × .04.

28. .0012 × .005; 2.26823 × 200; 5.6125 × .0768.

29. .7 × 7 × .07; .15625 × 23.7 × .00192 × 5.

30. (2.465 + 1.21) × (3.2 − 2.89).

31. $(3.01)^2$; $(.045)^2$; $(.0081)^2$; $(5.1004)^3$; $(.76)^3$.

32. $(.125)^2 \times (.32)^3$.

Divide:

33. 291.84 by 6; .12936 by 12; 7.92801 by .9.

34. 58.383 by .39; .28744 by .08; 491.205 by .065.

35. 68.325 by 6.25; .732 by 1.6; 1208.88 by .438.

36. 498 by .0125; 7 by .007; 1000 by .0001.

37. .235 by 10.24; 27 by 12; .00507702 by .0283.

38. 89.3 by .00752; 74.1 by .0256; 1 by .128.

39. .39842 by 3.7164; 281.5 by 13.789; .0005 by .0028.

40. 63.04128 by 912.85; 287.209 by .00493; 2000 by .0059.

Exercise II.

Find the second members of the following equations:

1. 1.4 + 2.08 + 3.895 =

2. 2.8 + 2.08 + .28 + .028 + .812 =

3. 1.667 + 0.4 + 0.286 + 6.08 + .636 + 0.931 =

4. 6.125 − .57 =

5. $(4.625 + 1.146) - (1.2 + 3.571) =$
6. $6.913 - (2.85 - 0.937) =$
7. $24 - 2.4 + (5 - 3.508) - 3.092 =$
8. $10 - (4.25 - 2.5 + 2 - 0.625 - 0.4 - 2.02) - 0.295 =$
9. $1.5 \times .08 \times .5 =$
10. $.1204 \times .0168 \times 100 =$
11. $.04 \times 3.25 \times .06 =$
12. $36 \times .002 \times 2.05 \times .00765 =$
13. $.139 \times 28 + 42 \times .002 + 6 \times .004 - .05 \times 20 =$

Exercise III.

1. What number subtracted 88 times from 80,005 will leave 13 as a remainder?
2. If 7 men can build a wall in 16 days, how many men will it take to build a wall three times as long in half the time?
3. How many minutes are there between 25 minutes past 8 in the morning and midnight?
4. The velocity of sound being 1090 feet per second, at what distance is a gun fired, the report of which I hear 11 seconds after seeing the flash? (5280 feet make a mile.)
5. How long would it take to travel 30.2375 miles at the rate of 8.85 miles per hour?
6. The circumference of a circle being 3.1416 times the diameter, find the circumference of a circle whose diameter is 6.8 feet; also, find the diameter of a circle whose circumference is 20 inches.
7. How much wire will be required to make a hoop 30 inches in diameter, allowing 2 inches for the joining?
8. How many times would such a hoop turn in going half a mile?

9. Cork, whose weight is .24 of that of water, weighs 15 pounds per cubic foot. What is the weight of 6 cubic feet of oak, the weight of oak being .934 of that of water?

10. From what number can 847 be subtracted 307 times, and leave a remainder of 49?

11. What is the 235th part of 141,235?

12. What will 343 barrels of flour cost, at $6.37 a barrel?

13. 12 make a dozen, and 12 dozen make a gross. How many steel pens in 28 gross? What will a gross of eggs cost, at 27 cents a dozen?

14. How much must be added to $4429 in order to make the sum 43 × $241?

15. What number deducted from the 26th part of 2262 will leave the 87th part of the same number?

16. At an ordinary rate, 123 words a minute, how long will it take a man to deliver a speech of 15 pages, each of 28 lines, and each line containing 11 words? How long would it have taken Daniel Webster to deliver the same speech, at the rate of 93 words a minute?

17. How long would it take a railway train to go from New York to San Francisco, 3310 miles, at the rate of 1973 feet a minute?

18. How long will it take to count a million, at the rate of 67 a minute?

19. If you put into a box 17 cents a day, including Sundays, beginning January 1 and ending July 4, how much money will there be in the box?

20. If a man's income is $3000 a year, and his daily expenses average $7.68, what does he save in a year?

21. In a question of division the quotient was 87.83, the divisor, 759. What was the dividend?

22. It is 3.1416 times as far round a wheel as across it. How many times will a wheel 4.5 feet across turn round in going 23 miles of 5280 feet each?

23. How many gallons of 231 cubic inches are contained in a cubic foot (1728 cubic inches)? in a bushel of 2150.42 cubic inches? How many cubic feet in a bushel? How many bushels in 31.5 gallons?

24. Seven children had left to them $7186 apiece; one died, and his share was divided among the surviving six. How much had each then?

25. What is the nearest number to 7196 that will contain 372 without a remainder?

26. How long will it take 2 men to do what 1 man can do in 6 days? what 4 men can do in 3 days? what 3 men can do in 4 days?

27. Divide $1.80 among Thomas, Richard, and Henry in such a way that Henry shall receive 3 cents for every 5 cents that Thomas gets, and Richard shall receive 2 cents for every 3 cents that Henry gets.

28. Divide $87.84 between B and C so that C shall get $19 as often as B gets $17.

29. Three partners received for goods: one, $371.63; the second, $285.40; the third, $411.91. They paid for the goods $879.34, and divided the balance equally among them. How much did each receive?

30. At 12 inches in a foot, how many inches long is a wall 35 feet in length? A brick and its share of mortar being 8.4 inches long, how many bricks in length is the wall?

31. A brick and mortar being 2.4 inches in height, how many bricks are required to build the wall 12 feet high, if the wall be two bricks wide.

32. What is the total weight of the wall, if a brick and its share of the mortar weigh 4.13 pounds? What is the weight after a long rain, when the weight is increased to 4.27 pounds for each brick?

33. How many pounds does each foot in length of the wall weigh?

34. If 60.98 cubic inches of brick weigh 4 pounds, how many cubic inches of brick weigh 1 pound? How many pounds would a cubic foot (1728 cubic inches) weigh?

35. If a cubic foot of water weigh 62.5 pounds, how many times as heavy as water is brick?

36. Light moves through the air at 186,500 miles in a second. How many times can it go around the earth in a second, the distance round the earth being 24,897.714 miles?

37. Light moves through the air at 300,190 kilometers in a second. How many times can it go around the earth in a second, the distance round the earth being 40,007.5 kilometers?

38. A minute is 60 seconds. How many miles and how many kilometers can light travel through air in a minute?

39. An hour is 60 minutes. How many miles and how many kilometers can light travel in an hour?

40. The distance round the earth, given in Ex. 37, is measured on a north and south line. Around the equator the distance is 40,075.45 kilometers. How many times could light move round the equator in one minute?

41. Find the reciprocal of the difference between 31.24 and 31.23768.

42. The Hanoverian mile is 25,400 Hanoverian feet long, each foot being .9542 of an English foot. Find to four places of decimals the fraction that an English mile of 5280 English feet is of a Hanoverian mile.

CHAPTER IX.*

MEASURES.

169. Continuous quantities (such as length, surface, bulk, value, heat) cannot be counted; they are measured.

170. To **measure** a quantity is to find how many times it contains a *known quantity* of the same kind, called the **unit of measure.**

171. The unit of measure for lengths is a **meter;** and from this are derived the units of surface, volume, and weight.

172. The meter was intended to be one ten-millionth of the distance from the equator to the north pole, but more careful measurements of meridians show that this distance is 10,001,887 meters.

173. The **standard meter,** as defined by law, is the length of a bar of very hard metal, carefully preserved at Paris, accurate copies of which are furnished to the governments of all civilized countries.

174. The principal units of measure are:

The meter (^{m}) for lengths;
The square meter (^{qm}) for surfaces;
The cubic meter (^{cbm}) for large volumes;
The liter (^{l}) (*lee'-ter*) for smaller volumes;
The gram (^{g}) for weights.

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* Chapters IX. and XIV. may be taken or omitted, at the option of the teacher.

175. All these units are divided and multiplied decimally, and the size of the measures thus produced is shown by one of seven prefixes; namely, *deka*, meaning 10; *hecto*, meaning 100; *kilo*, meaning 1000; *myria*, meaning 10,000; and *deci*, meaning 0.1; *centi*, meaning 0.01; *milli*, meaning 0.001.*

176. As in United States money we seldom speak of anything else than dollars and cents, so in other measures it is only those printed in **black letter** in this chapter that are in common use.

Money.

177. The unit of commercial values is the **dollar.** It is compared with the gram of gold by laws fixing the weight of gold which shall constitute a dollar; but these laws are changed from time to time. Coins are also made of silver, nickel, and bronze.

178. Divisions { A mill = 0.001 of a dollar.
A **cent** (ct.) = 0.01 " "
A dime = 0.1 " "
The unit, A **dollar** ($).
Multiple, An eagle = 10 dollars.

179. Cents are made of bronze; half-dimes of nickel; dimes, quarter-dollars, half-dollars, and dollars, of silver; quarter-eagles, half-eagles, eagles, and double eagles, of gold. But the eagle is usually called ten dollars, and the dime ten cents. That is to say, $137.875 is read 137 dollars and 87 and a half cents, and *not* 13 eagles 7 dollars 8 dimes 7 cents 5 mills.

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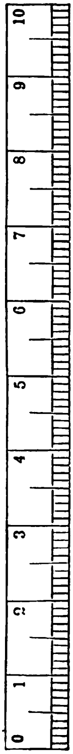
* All the compound names are accented on the first syllable, thus: mil'limeter.

Linear Measures.

180. Divisions
A **millimeter** (^{mm}) = .001 of a meter.
A **centimeter** (^{cm}) = .01 " "
A decimeter = .1 " "

The unit, A **meter** (^{m}).

Multiples
A dekameter = 10 meters.
A hektometer = 100 "
A **kilometer** (^{km}) = 1,000 "
A myriameter = 10,000 "



1 Decimeter = 10 Centimeters = 100 Millimeters.

181. A length given in any one of these measures may be expressed in terms of another measure by simply moving the decimal point to the right or left.

Thus, 17,856,342^{mm} may be written as **kilo**-meters by observing that **milli**-meters are changed to meters by moving the point three places to the left; and these meters into **kilo**-meters by carrying it three places further, making, in all, six places. Therefore, 17,856,342^{mm} = 17.856342^{km}.

Again, 4.876326^{km} may be written as centimeters, by observing that **kilo**-meters are changed to meters by moving the point three places to the right, and meters to **centi**-meters by moving it two places further, making, in all, five places. Therefore, 4.876326^{km} = 487,632.6^{cm}.

182. The rule, therefore, for this conversion is: First change the point so as to convert the **given measures** into terms of the **principal unit**; then change the point so as to convert the principal into the **required** units.

183. Remember that, before adding or subtracting, the quantities must be written in the **same units of measure.**

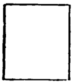
1. Convert 5427^{m} into kilometers; into millimeters; into centimeters.
2. 6853^{mm} contain how many meters? how many centimeters? what part of a kilometer?
3. Write 49.7^{m} as centimeters; as millimeters; as part of a kilometer.
4. How many centimeters in 12.4^{km}? how many millimeters?
5. Change 1230 meters into kilometers; into centimeters.
6. Write 1230^{cm} as meters; as millimeters.

Find the value of each of the following expressions in meters:

7. $.435^{m} + 852^{cm} + 4263^{mm} + .1595^{km}$.
8. $.927^{km} - 6495^{cm}$; $4.37^{cm} - 42.87^{mm}$.
9. $8 \times .0457^{km}$; 3.04×60.93^{cm}; 5.43×67.2^{mm}.
10. $38{,}019^{mm} \div .097$; $.41^{km} \div 25.625$.
11. At $1.87 the meter what is the cost of 6.20^{m} of cloth?
12. At $.75 the meter what is the cost of 60^{m} of cloth?
13. From a piece of cloth containing 47.60^{m} a tailor cuts off three pieces: the first of 3.80^{m}, the second of 1.30^{m}, and the third of 45^{cm}. How much of the cloth is left?
14. What is the value of 60^{cm} of cloth, worth $5.20 a meter?
15. If $6.00 are paid for a railroad ticket to travel 440^{km}, what is the fare per kilometer?
16. If a train run 288^{km} in 9 hours, how many meters does it run in a minute?
17. If a man walk at the rate of 6^{km} an hour, what part of an hour will it take to walk 420 meters?
18. A railroad carried 412 passengers 18 kilometers, and received $88.992; at the same rate, what will it receive for carrying 350 passengers 35 kilometers?

MEASURES OF SURFACE.

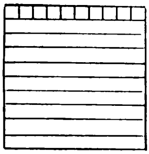
184. The unit of surface is a **square,** each side of which is a linear unit.



Square Centimeter.

185. The principal unit of surface is, therefore, a **square meter** (^{qm}).

186. But in square measure, the multiplication and division of units is by hundreds and hundredths, instead of by tens and tenths. Suppose the square in the margin to represent a **square meter.** It is divided into ten equal horizontal bands, and each band is one-tenth of the square meter. Each band can be divided, as the upper one is, into ten little squares measuring one-tenth of a meter on a side. Each of these squares will be .1 of the band, or .01 of the whole square. The **square meter,** therefore, contains 10 × 10 or 100 **square decimeters.**



187. If the square meter were divided into 100 equal horizontal bands, each band would be .01 of the square; and if each of the 100 bands were divided into 100 squares, that is, into 100 square centimeters, the whole square would contain 100 × 100 or 10,000 square centimeters. A **square meter,** therefore, contains 10,000 **square centimeters.**

188. If the square were divided into 1000 equal bands, each band would be .001 of the square; and if each of the 1000 bands were divided into 1000 squares, that is, into 1000 square millimeters, the whole square would contain 1000 × 1000 or 1,000,000 square millimeters. A **square meter,** therefore, contains 1,000,000 **square millimeters.**

That is,

189.

A square millimeter $(^{qmm})$ = .000001 of a square meter.
A **square centimeter** $(^{qcm})$ = .0001 " " "
A square decimeter = .01 " " "
A **square meter** $(^{qm})$ **Principal Unit.**
A square dekameter = 100 square meters.
A square hektometer = 10,000 " "
A **square kilometer** $(^{qkm})$ = 1,000,000 " "

It will be observed that while centimeters are in the second and millimeters in the third decimal place from meters, **square centimeters** are in the **fourth** and **square millimeters** in the **sixth** decimal place from **square meters.**

190. In the measurement of land, the **square dekameter** is called an **ar** $(^{a})$, and the **square hektometer** is called a **hektar** $(^{ha})$.

191. Be careful, in converting square measures from one unit to another, to observe that, when the **ar** is the unit, the point is to be moved as in linear measure; but when the **square meter** is the unit, the point is to be moved **twice as far** as in linear measure. Thus, it takes 100 **ars** to make a **hektar**; but 100 × 100, or 10,000 **square centimeters** to make a **square meter,** and 100 × 100, or 10,000 **square meters** to make a square hektometer, or **hektar.**

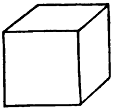
1. Convert $1{,}854{,}276^{qm}$ into hektars; into square kilometers.
2. How many hektars in 2.7856 square kilometers?
3. Write 1.7431^{qm} as square centimeters; as square millimeters.
4. How many square kilometers in 17,467.5 hektars?
5. How many square meters in 1.3614^{qkm}?
6. How many square meters in 2.25 hektars?

7. How many square centimeters in 0.0137 of a square meter?

8. Write 3.571^{qcm} as square millimeters.

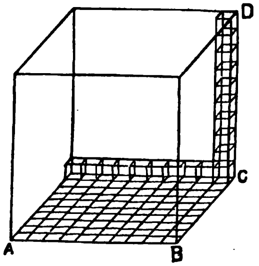
Measures of Volume.

192. The measure of capacity, or volume, is a cube, each face of which is a square unit.



Cubic Centimeter.

193. It is to be observed that the cubic meter can be divided into 10 layers, each a meter square and a decimeter thick. Each layer will, therefore, be .1 of a cubic meter.



Again, each layer can be divided into 10 equal parts. Each part will, therefore, be .1 of the layer, or .01 of the meter, and will be a decimeter square and a meter long.

Also, each one of these parts can be divided into 10 equal parts, each of which will be a cubic decimeter, and will be .1 of .01, or .001 of the cubic meter.

The **cubic meter,** therefore, contains 1000 **cubic decimeters.**

In like manner, each cubic decimeter can be divided into 1000 cubic centimeters, and each cubic centimeter into 1000 cubic millimeters.

That is,

194.

A cubic millimeter (^{cmm}) = 0.000000001 of a cubic meter.

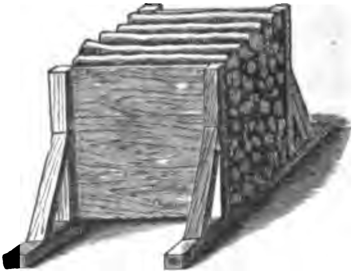
A cubic centimeter (^{ccm}) = 0.000001 " " "

A cubic decimeter = 0.001 " " "

A cubic meter (^{cbm}). **Principal unit.**

It will be seen that in cubic measure, the decimal point is to be moved **three times as far,** when changing the unit, as in linear measure.

1. How many cubic centimeters in 2.25^{cbm}?
2. How many cubic meters in $2,162,875^{ccm}$?

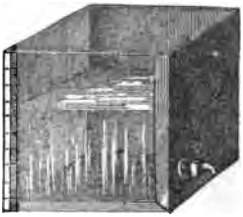


Ster of Wood.

195. In measuring wood the cubic meter is called a **ster** (st).

196. In measuring liquids, grain, etc., the **cubic decimeter** is always called a **liter.**

If the water in the liter represented in the margin stands 6^{cm} high, how many *cubic centimeters* of water are there in the measure? How many will be required to fill it? If the faucet be turned and the water allowed to run out until the measure is only half full, how many cubic centimeters will run out? How many will still remain?



Liter = Cubic Decimeter.

When the liter is the unit, the numeral prefixes have the same value as in linear measure. Thus,

197.

Divisions { A milliliter = .001 of a liter.
A centiliter = .01 " "
A deciliter = .1 " "

The unit, A **liter** $(^{l})$.

Multiples { A dekaliter = 10 liters.
A **hektoliter** $(^{hl})$ = 100 "
A kiloliter = 1000 "

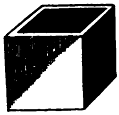
1. How many liters in 1.7^{cbm}? in $157{,}854^{ccm}$?

Liter (common form).

2. How many cubic centimeters in 9.5^{l}? in $.015^{l}$?
3. Change 1.25^{hl} to cubic centimeters; to the fraction of a cubic meter.
4. Convert 431.88^{l} into hektoliters; into the fraction of a cubic meter.
5. Write $.375^{cbm}$ as liters; as cubic centimeters.
6. Write $734{,}159.651^{ccm}$ as liters; as hektoliters; as cubic meters.
7. How many cubic meters in $8{,}573{,}412.867^{ccm}$?
8. Change the expression $.734578912^{cbm}$ into cubic centimeters; into liters.
9. Change 1731.5 liters into cubic meters; into cubic centimeters.

WEIGHTS.

198. The units of weight are the weights of units of pure water taken at its greatest density, that is, a little above the freezing point.



Cubic Centimeter.

Gram Weight.

The principal unit is the **gram**, which is the weight of a **cubic centimeter** of water.

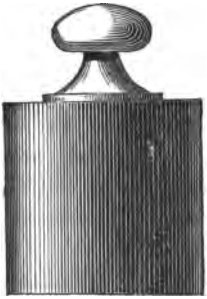
The numeral prefixes have the same value as in linear measure. Thus,

199. Divisions
A **milligram** (^{mg}) = 0.001 of a gram.
A centigram = 0.01 " "
A decigram = 0.1 " "

The unit, A **gram** (^{g}).

Multiples
A dekagram = 10 grams.
A hektogram = 100 "
A **kilogram** (^{kg}) = 1000 "
A **metric ton** (^{t}) = 1000 kilograms.

200. A **cubic centimeter** of water weighs a **gram.**
A **liter** of water weighs a **kilogram.**
A **cubic meter** of water weighs a **ton.**



Kilogram Weight.

1. How many kilos* in 1.73^{t}? in 0.341 of a ton?
2. How many kilos will a hektoliter of water weigh?
3. Convert 13,756^{mg} into grams; into the fraction of a kilo.
4. What is the weight in grams of 346.1^{ccm} of water?
5. Give the weight in kilograms of 0.37615^{cbm} of water.
6. Change .6778^{kg} into milligrams.
7. How many milligrams in the third part of 17.4 grams?

---

* Kilogram is generally called kilo.

EXERCISE IV.

1. Add 17.3^{m}, 87.41^{m}, 271^{cm}, 380^{mm}, and 1.79^{m}.
2. What is the sum of \$15.87, \$39.46, \$47.52, \$75.38, \$75.89?
3. Add 187^{cm}, 49.3^{m}, 317^{mm}, and 6.138^{m}.
4. The door-sill being 3^{cm} high; the door, 2.34^{m}; the finish over it, 13.7^{cm}; and the distance from finish to ceiling, 93^{cm}; how far from floor to ceiling?
5. The distance to the post-office is 3.31^{km}; thence to the mill, 1.711^{km}; thence to the store, 3.718^{km}; thence home, 2.543^{km}. How long is the circuit?
6. From Portland, Me., to Boston is about 132^{km}; Boston to Albany, 320^{km}; Albany to Buffalo, 480^{km}; Buffalo to Chicago, 800^{km}; Chicago to Omaha, 800^{km}; Omaha to Cheyenne, 780^{km}; how far from Cheyenne to Portland? to Albany? from Boston to Chicago? from Boston to Cheyenne?
7. If I travel 789.7^{km} a day, how far shall I go in 7 days? in 8.5? in 19.6? in 27.8? in 365 days?
8. How much will 3^{m} of cloth cost, at \$1.37 a meter? How much 5.38^{m}, at \$2.63 a meter?
9. How much will 13.4^{kg} of opium be worth, at \$8.48 a kilo? 28.79^{kg}, at \$7.96 a kilo?
10. A man bought 153 barrels of flour, at \$4.875 a bbl. What did the whole cost him?
11. He gave for it 6 shares of stock, at \$113.50 a share, and the rest in cash. How much money did he pay?
12. He paid \$13.75 for storage; also, 75 cents a barrel for freight. How much do these expenses amount to?
13. It was sold, 49 bbls. at \$6.50 a bbl.; the rest at \$6.25. What were the gross receipts?
14. He paid for commissions, etc., \$17.50; and counts his loss of interest at \$29.30. What then is his net profit?

201. *If the diameter of a circle be multiplied by* 3.1416, *the product is the length of the circumference.* (For great accuracy the diameter is multiplied by 3.1415927.)

15. Find the circumference of a circle having a diameter of 1^{m}.

16. Find the circumferences of circles of which the diameters are respectively 83^{m}; 3.71^{m}; 32.8^{m}; 10.4^{cm}; 11.8^{cm}; 167.1^{mm}; 39.3^{mm}. Give each to the nearest tenth of a millimeter.

17. What is the length of the earth's orbit, to the nearest meter, if the diameter of the orbit is $294{,}481{,}217^{km}$?

18. How far round this world, if its diameter is $12{,}734^{km}$?

19. If a carriage-wheel is 1.31^{m} in diameter, what is its circumference? How far will it go, if it roll without slipping, in turning once? 17 times?

20. How often must that wheel turn in going 69.429^{m}? 73.513^{m}? 17.27^{km}?

21. Find the reciprocal of 3.14159 to the 5th place.

202. *If, therefore, a circumference be multiplied by* .31831, *the product is the diameter.*

22. What is the diameter of the circle whose circumference is 314.159^{cm}?

23. What is the diameter of the wheel which revolves 19.5 times in going 107.25^{m}?

24. How thick through is a tree which has a girth of 2.97^{m}?

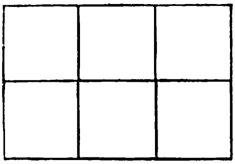
25. What is the diameter of a circular field two kilometers in circumference?

26. What is the diameter of a rope of which the circumference is 20^{cm}?

27. In a park is a fountain whose basin is 75^{m} in circumference. What is the diameter of the basin?

MEASURES OF SURFACE.

203. When a surface is flat, and has four square corners, it is called a rectangle. Suppose the rectangle in the margin is 3^{cm} long and 2^{cm} wide. If lines be drawn as represented in the figure, the surface will be divided into **square centimeters.** There will be 2 horizontal rows of 3 square centimeters each; that is, in all, 2 times 3 square centimeters. Hence,



Express the length and breadth of a rectangle in the **same linear unit**; *the product of these two numbers will be its area in* **square units** *of the same name as the linear unit of the sides.*

Conversely, the number of square units in a rectangle, divided by the number of linear units in one side, will give the number of linear units in the other side.

EXERCISE V.

1. Find the area of a rectangle 17^{cm} by 19^{cm}.
2. In a rectangular township 16^{km} by 7^{km}, how many hektars? If there are in it 47.3^{km} of highway, averaging 11.7^{m} wide; how much land is left for other uses?
3. In a rectangular field, 751.3^{m} long and 189.3^{m} wide, is a strawberry bed 31.4^{m} by 17.8^{m}. How many hektars in the field? How many, exclusive of the strawberry bed?
4. If my garden contain 941.65^{qm}, and my neighbor's 748.37^{qm}, what is the area of both in hektars?
5. If a painter can cover 8.786^{qm} in an hour, how much can he cover in 1.78 hours? in 3.86 hours? in 4.57 hours?

6. How many hektars in each of three rectangular fields: one measuring 315.71^{m} by 78.91^{m}; a second, 293.6^{m} by 84.84^{m}; the third, 346.8^{m} by 71.82^{m}. How many in the three?
7. Give the price of each field, and of the whole, at $67.50 a hektar; at $384 a hektar; and at $2.375 a square meter.

204. If the area of a square be multiplied by .7854 it gives the area of the largest circle which can be drawn in that square. In other words, *multiply the second power of the diameter by* .7854 (that is, by ¼ of 3.1416), *and the product is the area of the circle in* **square units** *of the same name as the* **linear unit** *of the diameter.*

8. What is the area of a circle 27^{cm} in diameter? of one which is 1^{m} in diameter?
9. What is the area in hektars of a circular field 784^{m} across?
10. Give the area of a circle 31^{cm} in diameter.
11. Find the length of a rectangle 17^{cm} wide, and containing 306^{qcm}. What length of carpet 75^{cm} wide is required to make 27^{qm}?
12. A room is 16^{m} long, 8^{m} wide, and 8^{m} high; another room is 7^{m} long, 7^{m} wide, and 3^{m} high. How many square meters of painting on the walls of both rooms, if no allowance is made for doors and windows? How many more square meters of painting on the walls of the larger room than on those of the smaller?

205. The surface of a sphere or globe is four times that of a circle of the same diameter. Therefore,

To find the surface of a sphere, multiply the square of the diameter by 3.1416.

13. How many square centimeters of surface on a ball 7^{cm} in diameter?
14. How many square centimeters of surface on a ball 18^{cm} in diameter?
15. How many square meters of surface on a hemispherical dome 11.27^{m} in diameter?
16. What is the interior surface of a hemispherical basin 12^{cm} in diameter?
17. What is the interior surface of a hemispherical vase 70^{cm} in diameter?

206. *Half the diameter is the radius; and* 3.1416 *times the square of the radius is the area of the circle.*

18. Find, by this rule, the area of example 9.
19. How many square centimeters are inclosed in a circle struck with a radius of 7^{cm}?
20. In a sheet of zinc 1.76^{m} long and 89^{cm} wide are two circular openings, one of which has a radius 10.5^{cm}, the other a radius 9.2^{cm}. What is the area of the zinc left?
21. What is the area of a circle of which the radius is 24^{m}?
22. A piece of land in the form of a circle has a radius of 40^{m}; in the middle of it is a pond forming a circle of 15^{m} radius. What is the total surface? the surface of the pond? the surface of the land to cultivate?

CARPETING ROOMS.

207. Carpeting is made of various widths and is sold by the length.

In determining the number of meters required for a room, we first decide whether the strips shall run lengthwise or across the room, and then find the number of strips needed. The number of meters in a strip multiplied by the number of strips will give the required number of meters, without allowance for waste in matching patterns.

23. How many meters of carpet 60^{cm} wide will be required for a room 6^{m} long and 5.4^{m} wide, the strips running lengthwise? how many meters would be required if the carpet were 80^{cm} wide?

Since the room is 540^{cm} wide, it will take $\frac{540}{60}$, or 9 widths of carpet 60^{cm} wide; that is, the amount required will be $9 \times 6^{m} = 54^{m}$. If the carpet were 80^{cm} wide, it would take $\frac{540}{80}$, or 7 widths;* that is, the amount required will be $7 \times 6^{m} = 42^{m}$.

24. How many meters of carpet 56^{cm} wide will be required for a room 8.32^{m} long and 6.6^{m} wide, strips running lengthwise?

25. How many meters of carpet 70^{cm} wide will be required for a room 7^{m} long and 5.4^{m} wide, strips running across the room?

26. How many meters of carpet 80^{cm} wide will be required for a room 6^{m} long and 5.47^{m} wide, strips running across the room?

27. How many meters of carpet 90^{cm} wide will be required for a room 5^{m} long and 4.5^{m} wide, strips running lengthwise? How much will it cost, at $1.875 a meter?

28. How many meters of carpet 75^{cm} wide will be required for a room 5.25^{m} long and 4.75^{m} wide, strips running across the room? How much will it cost, at $2.125 a meter?

29. How many meters of carpet 75^{cm} wide will be required for a room 5.6^{m} square? How wide a strip will have to be turned under? How much will the carpet cost, at $1.25 a meter?

---

* Six widths would leave a surface 60^{cm} wide to be covered. This surface would require another strip, of which a width of 20^{cm} would be "turned under." It is economy to select carpeting of the right width for the room.

PAPERING AND PLASTERING.

208. The area of the **four walls** of a room is equal **to that** of a rectangle whose length is the **perimeter** of the room, and whose breadth is the **height** of the room.

Perimeter = twice the length + twice the breadth.
Area = **perimeter × height.**

30. Find the **area** of the walls of a room whose length is 6.12^{m}, breadth 5.05^{m}, and height 3.5^{m}?

$$\text{Perimeter} = 2\,(6.12^{m} + 5.05^{m}) = 22.34^{m},$$
$$\text{Area} = 22.34^{m} \times 3.5^{m} = 78.19^{qm}.$$

31. How many rolls of paper 45^{cm} wide and 8^{m} long, allowing 11.19^{qm} for doors and windows, will be required to paper this room?

32. Find the cost of papering a room 8^{m} long, 5.5^{m} wide, and 4.5^{m} high, with paper 50^{cm} wide and 7.5^{m} in a roll, at $1.25 a roll, put on? There is a base-board 25^{cm} wide running round the room, and an allowance of 11^{qm} is made for doors and windows.

33. Find the cost of plastering this room, at $.50 a square meter.

34. Find the cost of papering a room 5.5^{m} long, 4.8^{m} wide, and 3.2^{m} high, with paper 45^{cm} wide, 7.5^{m} in a roll, at $.875 a roll, put on, allowing 12^{qm} for base-board, doors, etc.

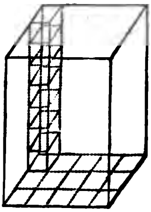
35. Find the cost of plastering this room, at $.45 a square meter.

36. Find the cost of papering a room 6^{m} square and 3.5^{m} high, with paper 45^{cm} wide and 7.5^{m} in a roll, at $.75 a roll, put on; and of putting on a border, at 5 cents per running meter.

37. Find the cost of plastering this room, at $.36 a square meter.

MEASURES OF VOLUME.

To find the volume of a rectangular solid.



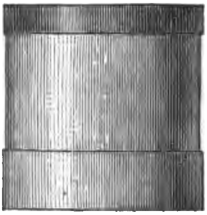
209. In the figure represented in the margin, let the length contain 5, the breadth 3, and the height 7 *linear centimeters.*

The base may be divided into square centimeters: there will be three rows of 5 square centimeters each; in all, 15 square centimeters. Upon each square centimeter may be placed a pile of 7 cubic centimeters; so that the solid will contain 15 × 7 cubic centimeters, that is, 3 × 5 × 7 *cubic centimeters.* Hence,

210. *Express the length, breadth, and height of a rectangular solid in the* **same linear unit**; *the product of these numbers will be its volume in* **cubic units** *of the same name as the linear unit of the edges.*

EXERCISE VI.

1. How many cubic centimeters in a block 9^{cm} long by 7^{cm} wide, and 6^{cm} deep?
2. If wood is cut into 120^{cm} lengths, and a pile is 43.7^{m} long and 1.4^{m} high, how many sters of wood are there in it?
3. In a grain elevator is a bin 11.2^{m} long, 4.34^{m} wide, and 2.83^{m} deep. How many hektoliters of grain will it hold?



Hektoliter.

4. If a liter of grain weigh .81 of the weight of a liter of water, how much will the grain in that bin weigh?

5. A bin measuring 16^m by 9.7^m, and 2.8^m deep, is full of oats, worth \$.98 a hektoliter. What is the whole worth?
6. A vat 197^{cm} long, 87^{cm} wide, and 63^{cm} deep, holds how many liters? What would be the weight of water required to fill it?
7. Add 1341^{ccm}, 231^l, and 2.13^{hl}, and give the sum in terms of each of the three units.
8. If a spring pours out 467.8^l each minute, how many hektoliters will it deliver in 60 minutes? in 37 minutes? in 78 minutes?
9. If 67.3^l of oil in a vat with perpendicular sides fill it to a depth of 173^{mm}, how deep will 13.7 times that quantity fill it? and how many hektoliters will there be?
10. Into a round cup 10^{cm} across, with perpendicular sides, pour oil until it is 1^{cm} deep; then there are 78.54^{ccm} of oil in the cup. (See § 204.) How many cubic centimeters will there be when the oil is 38^{mm} deep?
11. What is the capacity of a tin cup 95^{mm} across and 11.08^{cm} deep?
12. What are the capacities of two cylindrical vessels, one being 16.24^{cm} across and 19.95^{cm} deep, the other 75.4^{mm} across and 87.9^{mm} deep?

211. *The volume of a sphere is found by multiplying the cube of the diameter by* .5236.

13. How many cubic centimeters in a ball 10^{cm} in diameter? How much less if you take the more exact multiplier? § 201.
14. Into a cubical box 20^{cm} on a side, and full of water, an iron ball 20^{cm} in diameter is gently lowered until it touches bottom. How much water is left in the box? Answer in liters and in cubic centimeters.

15. One cask contains 171.4^{l} of oil; another, 209.3^{l}; a third, 73.8^{l}; while a square vat, 137^{cm} each way, is filled to a depth of 69^{cm}. How much oil in all the vessels? in liters and in hektoliters.

16. How many liters of air in a room 7.8^{m} long, 6.23^{m} wide, and 3^{m} high?

17. If a person's breathing spoil the air at the rate of $.2175^{cbm}$ a minute, how long will it take 3 persons sitting in the room, closed, to spoil the air?

18. How long, at the same rate, would the air in a hall 22^{m} long, 16^{m} wide, and 7^{m} high, last an audience of 280 persons?

19. How many cubic meters of wood in a round stick of equal size throughout, 37^{cm} in diameter and 8.4^{m} long?

EXERCISE VII.

1. What is the weight, in kilograms, of a hektoliter of water? of 73.8^{l} of water? of a cubic meter of water? of a cubic centimeter?

2. If a man buys half a ton of potatoes for $20, and retails them all, without waste, at 5 cents a kilogram, what profit does he make on the whole?

3. What is the weight of water required to fill a vat 98^{cm} long, 71^{cm} wide, and 38^{cm} deep?

4. If the vat of the last example were filled with brine weighing 1.04^{kg} to the liter, what would be the weight of the brine?

5. If the vat of Example 3 were filled with wine weighing $.981^{kg}$ to the liter, what would be its weight?

6. What is the total weight of 13 men averaging 73.48^{kg} each?

7. How many kilograms, and how many tons, would 3.6175^{cbm} of brick weigh, at 2 tons to a cubic meter? at 2.34 tons?
8. From a barrel containing 67^{kg} of granulated sugar were taken three parcels of 2.75^{kg} each, and four parcels of 7.50^{kg} each. How much is left in the barrel?
9. Into how many pills of 325^{mg} each can a mass of 7.8^{g} be divided?
10. A mass of 21.8^{g} is divided into 60 pills. What is the weight of each pill?

SPECIFIC GRAVITY.

212. *The* **specific gravity** *of any substance is* **the number of times** *the weight of the substance contains the weight of an equal bulk of water.*

Thus, if a sample of quicksilver has a specific gravity of 13.6, it is 13.6 times as heavy as water; a cubic centimeter of it would weigh 13.6^{g}; a liter of it would weigh 13.6^{kg}; and a cubic meter of it would weigh 13.6^{t}.

Again, if the specific gravity of a certain alcohol is .827, that is, if the alcohol weighs .827 as much as an equal bulk of water, then a cubic centimeter of it would weigh $.827^{\text{g}}$; a liter, $.827^{\text{kg}}$; and a cubic meter, $.827^{\text{t}}$.

213. Therefore, the **specific gravity** of a substance is the **number** that expresses the **weight** of **a cubic centimeter** of it in **grams**; or of a **liter** in **kilograms**; or of a **cubic meter** in **tons.**

214. When a substance heavier than water is under water, the water buoys it up just the amount of the weight of the water displaced by it. I weigh a lump of coal suspended by a thread; it weighs 1017^{g}. I then hang it, still suspended to the balance, in a pail of water, and lower it until the coal is covered with water; it now weighs 531^{g}, and has lost 486^{g}. The first information I get is, that the lump contains 486^{ccm} of coal, for it displaces 486^{ccm} of water. Then to obtain its specific gravity, that is, its weight compared with that of water as unity (§ 166), we have $1017 \div 486 = 2.092$ as the specific gravity of the piece of coal.

A piece of stone weighing 1.1^{kg} in air and $.6^{kg}$ in water, is bound by a thread to a block of wood; the two together weigh 1.28^{kg} in air and $.54^{kg}$ in water. What is the specific gravity of the wood?

The weight of the wood in air $= 1.28^{kg} - 1.1^{kg} = .18^{kg}$.

The weight of water displaced by stone and wood $= 1.28^{kg} - .54^{kg} = .74^{kg}$.

The weight of water displaced by stone alone $= 1.1^{kg} - .6^{kg} = .5^{kg}$.

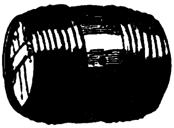
The weight of water displaced by wood, therefore, $= .74^{kg} - .5^{kg} = .24^{kg}$.

Hence, the specific gravity of the wood $= .18^{kg} \div .24^{kg} = .75$.

Ans. 0.75.

EXERCISE VIII.

1. If a stone weighs 1.3^{kg} in air and $.68^{kg}$ in water, and the the stone and a block of wood together weigh 1.55^{kg} in air and $.63^{kg}$ in water, what is the specific gravity of the block of wood?
2. What is the weight of 8.17^{hl} of alcohol, specific gravity .83?
3. What will 97^{l} alcohol weigh, of specific gravity .817? of specific gravity .819? of specific gravity .823? .838? .847?


Hektoliter.

4. A bar of aluminum 113^{mm} long, 17^{mm} wide, and 13^{mm} thick, is said to be of specific gravity 2.57. What does it weigh? If it really is of specific gravity 2.67, what does it weigh?

5. What would be the specific gravity of the bar of the last example if it weighed 65.137^{g}?
6. What is the weight of a bar of aluminum 371^{mm} by 63^{mm} by 84^{mm}, specific gravity being 2.63?
7. An irregular mass of copper, gently lowered into a pail brimful of water, caused 1.374^{l} to run over. What did it weigh if of specific gravity 8.91? if 8.89?
8. What was the specific gravity of that copper if the mass weighed 12.3016^{kg}?

215. The pupil will have seen, from the preceding examples, that to find the specific gravity of a body he may

Divide the **weight in grams** by the **bulk in cubic centimeters,**
the **weight in kilograms** by the **bulk in liters,** or
the **weight in tons** by the **bulk in cubic meters.**

9. A plate of iron 137^{cm} long, 64.3^{cm} wide, and 4.31^{cm} thick, weighs 277.54^{kg}. What is its specific gravity? What would the same mass weigh at specific gravity 7.47? at 7.79?
10. What is the specific gravity of sea-water when a hektoliter weighs 102.58^{kg}? what when 3^{l} weigh 3077^{g}?
11. What is the specific gravity of a substance of which 7.3^{ccm} weighs 31.5^{g}?
12. If a cubic meter of sand weighs 1723^{kg}, what is its specific gravity? If 3.4^{cbm} of gravel weigh 7.134 tons, what is the specific gravity?
13. If a cubic centimeter of metal weighs 7.3^{g}, what is its specific gravity?
14. What is the specific gravity of a fluid weighing 2.317^{kg} to a liter?
15. If a body weigh 3.71^{kg} in air and 2.38^{kg} in water, what is its specific gravity?
16. A piece of ore weighing 3.77^{kg} weighs in water only 2.53^{kg}. What is its specific gravity?

17. How many cubic centimeters in a stone which loses 17.8^{g} of its weight when weighed in water? What is its specific gravity if weighed in air it weighs 33.7^{g}?

18. In a wrought-iron bottle I find 2.63^{l} of quicksilver, weighing 35.81^{kg}; in another 2.59^{l}, weighing 35.193^{kg}; in a third, 2.617^{l}, weighing 35.571^{kg}. What is the specific gravity of each? What would be the specific gravity if the three were emptied into one vessel and mixed?

19. A plate of iron 89^{cm} by 17^{cm} by 7^{cm} weighs 79.43^{kg}. What is its specific gravity?

Exercise IX.

1. If three men eat 8^{kg} a week, how much would one man eat at the same rate? How much would seven men? At the same rate, how much do the three men eat in one day? and how much each man? At the same rate, how much would seven men eat each day? each week? in 5 weeks?

2. At the same rate, how much would 17 men eat in 3 weeks and 4 days?

3. If one hektoliter of oats is enough for 5 horses one week, how much is enough for 1 horse one week? for 1 horse 7 weeks? for 11 horses 17 weeks?

4. If two hektoliters of grain are enough for 3 horses 5 days, how much is enough for 3 horses 1 day? for 1 horse 1 day? for 7 horses 6 days?

5. Mix 17 liters of vinegar, costing 6 cents a liter, with 39^{l} at 5 cents, 21^{l} at 7 cents, and 13^{l} of water costing nothing. Find the amount of the mixture, and its cost?

6. For how much a liter must I sell that mixture, in order to gain 96 cents? for how much to clear $1.41?

7. A grocer sold 421 kegs of butter for $4995.25; 56 kegs brought $12.50 a keg; 91 brought $11.75 a keg; and 100 kegs brought $12.25 a keg. For how much a keg were the other kegs sold?

8. If 3 tons of coal cost $15.75, how many tons will $36.75 buy?

9. If 5^{m} of cloth cost $18.75, what should 7^{m} cost?

10. If a tap running 3.5^{l} a minute fills a tub in 16 minutes, how long should a tap delivering 5^{l} a minute be in filling the same tub?

11. If both taps of the last example be opened at once, how soon will they fill the tub?

12. If 3 men can dig 378^{m} of ditch in 2 days, how long will it take 5 men, at the same rate, to dig 787^{m}?

13. Into a tub which will hold 48^{l}, one tap is delivering water at the rate of 3.7^{l} a minute; while out of it, by another tap, the water is running at 2.5^{l} a minute. How long will it take to fill the tub, beginning with it empty?

14. A tap discharges into a tub 4.2^{l} a minute; from the tub water is also running, by a second tap; the water in the tub gains 30^{l} in 18 minutes. How fast is the second tap discharging?

15. If a wheel is 1.2^{m} across, how often will it turn in going one kilometer?

16. How many times in a minute does the wheel of the last example turn, when the carriage is driven 14^{km} an hour?

17. What is the weight of the water in a tank if it would take a flow of 8.7^{l} a minute 1 hour and 38 minutes to empty it?

18. Replace that bulk of water with oil worth $18.75 a hektoliter, and what will the contents of the tank be worth?

Exercise X.

1. A train leaves Paris at 11 o'clock A.M., and reaches Lyons at 10 o'clock P.M. How many meters does it travel in an hour, the distance from Paris to Lyons being 512.7^{km}?
2. A railroad has a single track 11.450^{km} long. How many rails 4.569^{m} in length did it require to lay the track?
3. A book is 2.1^{cm} in thickness; each leaf is $.05^{mm}$ thick. Find the number of pages in the book.
4. The cost of opening a canal amounts to \$25,400 a kilometer. How much would a canal cost which was 113.253^{km} in length?
5. The expense of laying out a paved road is \$12,500 a kilometer. How much would a road cost which was 72.053^{km} long?
6. The cost of building a railroad is about \$78,000 a kilometer in France, and only \$25,000 in the United States. How much would it cost in each country to make a road 295.671^{km} long?
7. If you must go up 211 steps to reach the top of a tower, and each step is 195^{mm} high, what is the height of the tower?
8. A house has 5 stories, each story has 19 stairs, each stair is 16^{cm} in height. Calculate how high the floor of the fifth story is from the ground.
9. A ream of paper contains 20 quires, each quire has 24 sheets, the ream is 13.5^{cm} in thickness. Find the thickness of each sheet.
10. The equator on a terrestrial globe measures $.80^{m}$ in circumference. By the aid of a tape-measure we find that the distance between two cities on this globe is $.046^{m}$. What is really the distance in kilometers between the two cities? (The earth's equator is $40{,}075.45^{km}$.)

11. Upon a military map we find that the distance from Paris to St. Denis is 78^{mm}. What is the distance in kilometers from Paris to St. Denis? The map is made on the scale of 1 to 80,000; that is, 1^{m} on the map represents $80{,}000^{m}$ of actual measurement upon the ground.

12. Give the number of revolutions made by the wheels of a carriage in travelling 82^{km}. The wheels are 1354^{mm} in diameter.

13. How many hektars in a square kilometer? how many ars? how many square meters?

14. France has about $542{,}000^{qkm}$. How many hektars does it measure?

15. A piece of land 1224.5^{m} square is sold at $140 a hektar. How much does the land bring?

16. The total surface measurement of the glass in the windows of a house is 182^{qm}. How many panes of 53^{cm} by 48^{cm} will it take to supply the windows?

17. How many square slabs of marble 150^{qcm} on the surface will it require to pave a court whose area is 25.35^{qm}?

18. A speculator bought 31.0728^{ha} of land for $1,296 a hektar. For how much a square meter must he sell it to realize a profit of $1,937?

19. A man is offered $6,000 for 2.5 ars of land. He declines to sell; and soon after, the town gives him $25.20 a square meter. How much did he make by refusing the first offer?

20. A man surveys a piece of land and finds that it measures 14.0715^{ha}. He afterwards discovers that his chain was too short by $.03^{m}$. How can he calculate the real superficial measurement of his land without surveying it again? (A surveyor's chain is 10^{m} long.)

21. The railroad from Paris to Orleans has a double track; each rail is 4^{m} long, and the distance from Paris to Orleans is 121^{km}. What is the number of rails used in laying the track? The width of the road is 15^{m}; how many hektars of land does the road include?

22. Calculate the number of ars in a surface which a ream of paper (480 sheets) will cover. The sheets are 30.3^{cm} long and 195^{mm} wide.

23. A pile of wood is 4.25^{m} long, 1.33^{m} thick, and 2.60^{m} high. How many sters are there in it?

24. A beam is 7.070^{m} long; its two other dimensions are $.258^{m}$ and 87^{mm}. Find its volume.

25. A bar of iron 3^{m} long measures 45^{mm} square on the end where it has been evenly cut. The bar is heated and drawn out to a greater length by being passed through an orifice 24^{mm} square. What is the length of the bar after the operation?

26. A reservoir is 1.50^{m} wide, 2.80^{m} long, and 1.25^{m} deep. Find how many liters it contains when full, and to what height it would be necessary to raise it that it might contain 10^{cbm}?

27. Suppose a box to be 3.75^{m} long, 3.50^{m} wide, and $.50^{m}$ high. How much lime would it take to fill it with mortar, reckoning that 1^{cbm} of lime after being slaked becomes 1.80^{cbm} of mortar?

28. A chest has the following dimensions: 1.17^{m}, $.90^{m}$, 1.04^{m}. How many cakes of soap 13^{cm} square on the bottom and 29^{cm} thick could be put in it? .12 of the volume of the chest must be deducted for packing.

29. A cubic meter of dry plaster makes 1.18^{cbm} when tempered; tempered plaster increases 1 in every 100, twenty-four hours after it is mixed. What volume of tempered plaster would be obtained from 55 sacks of 25^{l} each of dry plaster?

CHAPTER X.

MULTIPLES AND MEASURES OF NUMBERS.

216. If one number can be divided by another number, *without remainder*, the divisor is called a **factor,** or **measure,** of the dividend, and the dividend a **multiple** of the divisor.

Thus, 35 can be divided by 5 without remainder; therefore 5 is called a factor or measure, of 35, and 35 a multiple of 5.

217. Numbers which can be divided, without remainder, only by themselves and 1, are called **prime numbers.**

The smaller ones are easily found by trial, such as 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, etc.

218. Other numbers are each the product of a **fixed set** of prime factors, and are called **composite numbers.**

219. Numbers divisible by 2 are called **even numbers.** All other numbers are **odd numbers.** All primes (except 2) are odd.

220. Write a series of natural numbers in order; cancel the even ones; then place a dot over the multiples of 3: you produce this result:

$$1,\ \cancel{2},\ \dot{3},\ \cancel{4},\ 5,\ \cancel{\dot{6}},\ 7,\ \cancel{8},\ \dot{9},\ \cancel{10},\ 11,\ \cancel{\dot{12}},\ 13,\ \cancel{14},\ \dot{15},\ \text{etc.}$$

Each multiple of 6 has both the dot and the cancelling line, and the only numbers without the dot or line come just before or just after the multiples of 6. Therefore,

If a prime number be divided by 6 *the remainder must be either* 1 *or* 5.

221. Find the prime factors of 72.

$$\begin{array}{r|r} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

That is, $72 = 2 \times 2 \times 2 \times 3 \times 3,$

$= 2^3 \times 3^2.$

Hence, to separate a composite number into its prime factors,

Divide the given number by any prime number that is contained in it without remainder, then the quotient by any prime number that is contained in it without remainder, and so on until the quotient is itself a prime number. The several divisors and the last quotient are the prime factors required.

222. The following tests are very useful for determining without actual division whether a number contains certain factors:

1. A number is divisible by 2 if its *last digit is even.*
2. A number is divisible by 4 (2^2) if the number denoted by the *last two digits* is divisible by 4.
3. A number is divisible by 8 (2^3) if the number denoted by the *last three digits* is divisible by 8.
4. A number is divisible by 3 if the *sum of its digits* is divisible by 3.
5. A number is divisible by 9 (3^2) if the *sum of its digits* is divisible by 9.
6. A number is divisible by 5 if its *last digit* is either 5 or 0.

7. A number is divisible by 25 (5^2) if the number denoted by the *last two digits* is divisible by 25.

8. A number is divisible by 125 (5^3) if the number denoted by the *last three digits* is divisible by 125.

9. A number is divisible by 6 if its *last digit is even* and *the sum of its digits* is divisible by 3.

10. A number is divisible by 11 if the *difference between the sum of the digits in the even places and the sum of the digits in the odd places* is either 0 or a multiple of 11.

NOTE. The shortest method of dividing by 25 is to multiply by 4 and divide by 100; by 125, is to multiply by 8 and divide by 1000.

In adding the digits of a number to determine whether their sum is a multiple of a certain number, omit those digits which are seen at a glance to be multiples of the number. Thus, to discover whether 8,983,167 is divisible by 3, omit 9, 3, 6 (8, 1), which are manifestly multiples of 3, and simply add 8 and 7.

223. Other prime factors, 7, 13, 17, 19, sometimes betray their presence to one familiar with the subject; but, practically, the best way to detect them is to attempt to divide by them.

224. If we divide any number less than 121 (11^2) by 11, or by a number greater than 11, it is plain that the quotient is less than 11.

If we divide any number between 121 and 143 (11×13) by 11, the quotient will evidently lie between 11 and 13; and, since there are no prime numbers between 11 and 13, the quotient, if a whole number, must be composite, and contain factors smaller than 11.

What is thus proved of 11 and 13 is evidently true of any two *adjacent* prime numbers; namely, that, excepting the *second power* of the smaller prime number, *every composite number less than the product of two adjacent prime numbers, contains prime factors less than the smaller of these two numbers.*

Thus, every composite number less than 4087 (61 × 67), except 3721 (61^2), contains prime factors less than 61.

225. From the preceding article, the value of the following table, in discovering the prime factors of a given number, will be apparent.

Primes . .	7	11	13	17	19	23	29	31	37
Powers . .	49	121	169	289	361	529	841	961	1369
Products .	77	143	221	323	437	667	899	1147	1517

Primes . .	41	43	47	53	59	61	67	71	73
Powers . .	1681	1849	2209	2809	3481	3721	4489	5041	5329
Products .	1763	2021	2491	3127	3599	4087	4757	5183	5767

Primes . .	79	83	89	97	101	103	107	109	113
Powers . .	6241	6889	7921	9409	10201	10609	11449	11881	12769
Products .	6557	7387	8633	9797	10403	11021	11663	12317	14351

Opposite to "Powers" are placed the squares of the primes from 7 to 109; and opposite to "Products" are placed the products of the successive pairs of adjacent primes from 7 to 113.

226. Find the prime factors of 610,764.

$$\begin{array}{r|r} 2^2 & 610,764 \\ \hline 3 & 152,691 \\ \hline 7 & 50,897 \\ \hline 11 & 7,271 \\ \hline & 661 \end{array}$$

As 64 is divisible by 4, but 764 is not divisible by 8, 2^2 is the highest power of 2 contained in 610,764.

As the sum of the digits 152,691 is divisible by 3 but not by 9, 3^1 is the highest power of 3 contained in 152,691.

The next quotient, 50,897, does not contain 5; but divided by 7 gives 7271. 7271 does not contain 7; but, since $7 + 7 - (2 + 1) = 11$, it is divisible by 11.

The quotient 661 when divided by 6 gives a remainder of 1, which shows that it *may be* a prime number. It cannot be divided by 11, 13, 17, or 19, and is seen by the table to be less than $667 = (23 \times 29)$, and not equal to 529 (23^2); therefore it is a prime number.

Thus, $610,764 = 2^2 \times 3 \times 7 \times 11 \times 661$.

EXERCISE XI.

Find the prime factors of:

1. 148; 264; 178; 183; 173; 187; 346; 343;
2. 210; 353; 5280; 231; 31,416; 1369; 1368;
3. 247; 327; 179; 83; 2125; 2353; 2333;
4. 165; 168; 2148; 16,662; 321; 1551; 38;
5. 82; 129; 72; 66; 68; 65; 76; 86; 88; 142;
6. 326; 368; 464; 292; 362; 365; 730; 42;
7. 868; 999; 822; 1346; 7641; 6234; 234;
8. 579; 577; 212; 126; 128; 8192; 8190;
9. 8197; 3125; 2401; 1331; 78,309; 25,179.

227. A number is not only divisible by each of its prime factors, but by every possible combination of them. For example, 120 is $2^3 \times 3 \times 5$, and is divisible either by 2, 4, 8, 6, 12, 24, 30, 60, 10, 20, 40, or 15.

228. The number 14.21 may be put in the form of $1421 \times .01$; and be thus resolved into $7^2 \times 29 \times .01$. But .01 is not properly a factor, it is a divisor; it is the reciprocal of $2^2 \times 5^2$. Nevertheless, it is frequently of great practical advantage to separate mixed decimals, in this way, by first taking out the apparent factors .1, .01, .001, etc. Thus, the factors of 142.1 may be said to be 7, 7, 29, and .1; of 1.421, 7, 7, 29, and .001.

EXERCISE XII.

Find the prime factors of:

1. 8.4; 7.6; 1.08; .144; .036; .037; 21.45;
2. 14.6; 2.61; 21.2; 78.54; .5236; .00052;
3. 8.67; 48.3; 99.99; 5.04; 1.485; .216;
4. 34.87; 32.4; 5.115; 71.2; 2.993.

GREATEST COMMON MEASURE.

229. The measures of 30 and 50 respectively are: 1, 2, 3, 5, 6, 10, 15, 30; 1, 2, 5, 10, 25, 50. It will be seen that these two numbers have the measures 1, 2, 5, 10 in common, and of these measures 10 is the greatest.

230. The measures that two or more numbers have in common are called their *common measures*, and the greatest of these is called their **Greatest Common Measure,** which for the sake of brevity is indicated by the letters G. C. M.

231. If two or more numbers have no common measure except unity, they are said to be *prime to each other*. Thus 8 and 27 are prime to each other.

232. The prime factors of 30 are 2, 3, 5.
The prime factors of 50 are 2, 5^2.
The prime factors *common* to 30 and 50 are 2, 5.
The G. C. M. of 20 and 30, namely 10, is 2 × 5.

That is,

The G. C. M. *of two or more numbers consists of the* **prime factors common to the numbers,** *each prime factor having the* **lowest exponent** *that it has in any one of the numbers.*

233. Hence, to find the G. C. M. of two or more numbers,

Separate the numbers into their prime factors.

Select the **lowest power** *of each factor that is common to the given numbers, and find the product of these powers.*

Find the G. C. M. of 108, 396, 1440.

$$\begin{array}{r|r} 2^2 & 108 \\ \hline 3^2 & 27 \\ \hline & 3 \end{array} \qquad \begin{array}{r|r} 2^2 & 396 \\ \hline 3^2 & 99 \\ \hline & 11 \end{array} \qquad \begin{array}{r|r} 2^3 & 1440 \\ \hline 2^2 & 180 \\ \hline 3^2 & 45 \\ \hline & 5 \end{array}$$

Hence, the G. C. M. $= 2^2 \times 3^2$, or 36.

234. The factors that are common to two or more numbers may be taken out of the numbers simultaneously, as follows:

$$\begin{array}{r|rrr} 2^2 & 108 & 396 & 1440 \\ \hline 3^2 & 27 & 99 & 360 \\ \hline & 3 & 11 & 40 \end{array}$$

By the tests given in § 222, 4 (2^2) is seen to be common to the numbers, and 9 (3^2) common to the resulting quotients; the quotients 3, 11, 40, have no common factor, therefore 2^2 and 3^2 are the only factors common to the numbers. Hence, their G. C. M. is $2^2 \times 3^2$, or 36.

Exercise XIII.

Find the G. C. M. of

1. 27 and 33.
2. 13 and 39.
3. 8 and 28.
4. 27 and 45.
5. 81 and 108.
6. 4, 10, 12.
7. 4, 6, 10.
8. 9, 12, 21.
9. 10, 15, 25.
10. 14, 98, 42.
11. 30, 18, 54.
12. 14, 56, 42.
13. 96, 36, 48.
14. 84, 105, 63.
15. 24, 60, 84, 128.
16. 45, 81, 27, 90.
17. 78, 18, 54, 42.
18. 98, 28, 70, 42.
19. 96, 112, 80, 32.
20. 24, 96, 48, 120.
21. 84, 252, 168, 210.
22. 33, 88, 77, 55.
23. 252, 315, 420, 504.
24. 128, 192, 320, 368, 432.
25. 136, 204, 357, 459.
26. 909, 1414, 2323, 4242.

235. When it is required to find the G. C. M. of two or more numbers which cannot readily be resolved into factors, the method to be employed is as follows:

Find the G. C. M. of 18 and 48.

$$\begin{array}{l}18)48(2\\ \quad 36\\ \quad \overline{12})18(1\\ \qquad\ 12\\ \qquad\ \overline{\ 6})12(2\\ \qquad\qquad \underline{12}\end{array}$$

6 is the G. C. M. required.

This method depends upon two principles:

1. That every factor of a number is also a factor of every multiple of that number. Thus 4, which is a factor of 12, is a factor also of 24, 36, etc.

2. That any common factor of two numbers is also a factor of their sum and of their difference. Thus 4, which is a common factor of 24 and 36, is also a factor of 60 and 12.

Apply these principles to this example:

Since 6 is a factor of itself and of 12, it is, by (2), a factor of 18.

Since 6 is a factor of 18, it is, by (1), a factor of 2 × 18, or 36; and therefore, by (2), it is a factor of 36 + 12, or 48.

Hence, 6 is a common factor of 18 and 48.

Again, every common factor of 18 and 48 is, by (1), a factor of 2 × 18, or 36; and, by (2), a factor of 48 – 36, or 12.

Every such factor, being now a common factor of 18 and 12 is, by (2), a factor of 18 – 12, or 6.

Therefore, the **greatest** common factor of 18 and 48 is contained in 6, and cannot be greater than 6. And 6, which has been shown to be a common factor of 18 and 48, must be their G. C. M.

236. It will be seen that every remainder in the course of the operation contains, as a factor of itself, the G. C. M. sought; and that this is the greatest factor common to that remainder and the preceding divisor.

Therefore, a factor which is discovered, at any stage of the process, to belong to one of the numbers and not to the other, may be ejected; and a factor which is discovered, at any stage of the process, to belong to both numbers, may be taken out and reserved as a factor of the G. C. M.

(1) Find the G. C. M. of 11,237 and 12,559.

$$\begin{array}{r}
11237)12559(1 \\
11237 \\
2)\overline{1322} \\
\overline{661})11237(17 \\
\underline{661} \\
4627 \\
\underline{4627}
\end{array}$$

Hence, the G. C. M. is 661.

The factor 2 is thrown out of the first remainder 1322, for it is not contained in 11,237, and therefore is not a factor of the G. C. M. sought.

(2) Find the G. C. M. of 269,178 and 352,002.

$$\begin{array}{r}
6\,\underline{|\;269178 \quad 352002} \\
44863\;)\;58667(1 \\
44863 \\
4)\overline{13804} \\
3451)44863(13 \\
\underline{3451} \\
10353 \\
\underline{10353}
\end{array}$$

Hence, the G.C.M. is $6 \times 3451 = 20{,}706$.

The common factor 6 is first taken out from both numbers. From the remainder 13,804 the factor 4, which is prime to 44,863, is ejected. The resulting number 3451 is contained 13 times in 44,863, and therefore the G.C.M. is $6 \times 3451 = 20{,}706$.

EXERCISE XIV.

Find the G. C. M. of

1. 2,479 and 3,589.
2. 3,045 and 6,195.
3. 568 and 712.
4. 11,023 and 6,493.
5. 1,485 and 2,160.
6. 7,040 and 7,392.
7. 2,760 and 4,485.
8. 1,177 and 2,675.
9. 78,473 and 94,653.
10. 35,143 and 10,283.
11. 44,323 and 61,087.
12. 232,353 and 39,699.
13. 33,853 and 35,017.
14. 5,115 and 7,254.
15. 2,268 and 3,348.
16. 1,003 and 2,419.
17. 419 and 52,301.
18. 30,072 and 133,784.
19. 4,257 and 10,836.
20. 17,104 and 27,794.

237. To find the G. C. M. of several large numbers:

Find the G. C. M. of two of the numbers; then of that result and a third number; then of that result and a fourth; and so on. The last G. C. M. is the one required.

The work can generally be very much shortened by removing from each of the numbers all factors less than 13. Of these, the factors common to the numbers must be retained as factors of the G. C. M.

Find the G. C. M. of 3555, 4977, and 6636.

$$\begin{array}{rrrrrr} 3 & 3555 & & 4977 & & 6636 \\ \hline 5) & 1185 & 3) & 1659 & 4) & 2212 \\ 3) & 237 & 7) & 553 & 7) & 553 \\ \hline & 79 & & 79 & & 79 \end{array}$$

Hence, the G. C. M. is 3 × 79, or 237.

The common factor 3 is first taken out and reserved as a factor of the G. C. M. From the resulting quotients the factors less than 13 are removed, and 79 is found to be common to the numbers. Hence, their G. C. M. is 3 × 79.

Exercise XV.

Find the G. C. M. of

1. 855, 1,197, 1,596.
2. 3,864, 3,404, 3,657.
3. 15,561, 11,115, 13,585.
4. 2,943, 2,616, 4,578.
5. 1,177, 1,391, 1,819.
6. 4,939, 1,347, 3,143.
7. 740, 333, 296.
8. 833, 1,785, 1,309.
9. 7,326, 8,547, 9,768, 22,755.
10. 4,994, 7,491, 9,988, 12,485, 16,571.

Least Common Multiple.

238. The multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, etc.

The multiples of 3 are 3, 6, 9, 12, 15, 18, etc.

The multiples common to 2 and 3 are seen to be 6, 12, 18, etc.; and of these, 6 is the least.

239. The multiples that two or more numbers have in common are called their *common multiples,* and the least of these is called their **Least Common Multiple,** which is indicated by the letters L. C. M.

Find the L. C. M. of 7, 8, 9, 10.

The L. C. M. of 7, 8, 9, 10 must contain the factor 7, else it would not be a multiple of 7. It must also contain 2^3 to be a multiple of 8, and 3^2 to be a multiple of 9. It must contain the factors 2 and 5 to be a multiple of 10. That is, the L. C. M. of 7, 8, 9, 10, must contain the factors 7, 2^3, 3^2, and 5; therefore it is $7 \times 2^3 \times 3^2 \times 5$, or 2520. Hence,

240. To find the L. C. M. of two or more numbers:

Separate each number into its prime factors.

Select from these the **highest power** *of each.*

Find the product of these powers.

Find the L. C. M. of 14, 18, 21, 27, 28, 126.

$$\begin{aligned} 14 &= 2 \times 7, \\ 18 &= 2 \times 3^2, \\ 21 &= 3 \times 7, \\ 27 &= 3^3, \\ 28 &= 2^2 \times 7, \\ 126 &= 2 \times 3^2 \times 7. \end{aligned}$$

Hence, the L. C. M. $= 2^2 \times 3^3 \times 7 = 756.$

241. The L. C. M. of 14, 18, 21, 27, 28, 126, may be found as follows:

$$\begin{array}{r|rrrrrr}
2 & \cancel{14} & \cancel{18} & \cancel{21} & 27 & 28 & 126 \\ \hline
2 & & & & 27 & 14 & 63 \\ \hline
3 & & & & 27 & \cancel{7} & 63 \\ \hline
3 & & & & 9 & & 21 \\ \hline
 & & & & 3 & & 7
\end{array}$$

Hence, the L. C. M. $= 2^2 \times 3^3 \times 7$, or 756.

Since 14 is contained in 28, it is omitted, for any multiple of 28 is also a multiple of 14. Likewise 18, which is contained in 126, and 21, which is also contained in 126, are omitted.

The even numbers are divided by 2; the quotients and the odd numbers are written below the horizontal line.

The operation of dividing by 2 is repeated; the quotients and the odd numbers are written below.

Of the resulting numbers (27, 7, 63) 7, which is contained in 63, is omitted, and division by 3 gives the quotients 9 and 21.

The second division by 3 gives the quotients 3 and 7, which are seen to be prime to each other.

By this process the prime factors of each of the given numbers are obtained as divisors or as last quotients; therefore, the product of the divisors and last quotients is the L. C. M. required.

EXERCISE XVI.

Find the L. C. M. of

1. 6, 14, 21.
2. 8, 12, 3, 24.
3. 6, 10, 15,
4. 9, 12, 18, 4.
5. 15, 21, 35.
6. 12, 20, 24.
7. 14, 24, 28.
8. 12, 15, 20.
9. 16, 24, 32.
10. 21, 33, 77.
11. 27, 33, 99.
12. 7, 11, 13.
13. 77, 55, 35.
14. 16, 18, 27, 72.
15. 10, 12, 22, 33, 60.
16. 15, 16, 18, 20, 22, 24.
17. 56, 64, 70, 84, 112.
18. 48, 54, 81, 144, 162.
19. 75, 100, 120, 150, 180.
20. 112, 168, 196, 224.
21. 7, 14, 15, 21, 45.
22. 16, 25, 81.
23. 26, 39, 52, 65.
24. 80, 72, 225, 48.
25. 10, 20, 30, 40, 50, 60.
26. 30, 42, 105, 70.
27. 36, 24, 35, 20.
28. 7, 11, 14, 15.
29. 12, 18, 27, 63, 28.
30. 34, 26, 65, 85, 51, 39.
31. 12, 18, 96, 144.
32. 84, 156, 63, 99.
33. 17, 51, 119, 210.
34. 16, 30, 48, 56, 72.
35. 27, 33, 54, 69, 132.
36. 15, 26, 39, 65, 180.
37. 44, 126, 198, 280, 330.
38. 50, 338, 675, 975.
39. 552, 575, 920.
40. 228, 304, 342.
41. 1,080 and 1,260.
42. 600 and 480.
43. 1,564 and 1,932.
44. 2,530 and 1,760.
45. 936 and 2,925.
46. 3,432 and 4,032.
47. 1,875 and 2,425.
48. 1,632 and 2,976.
49. 1,001 and 2,233.
50. 539 and 1,463.

242. If the given numbers are large and contain no prime factors that can readily be detected, it is best to obtain the common factors by the process for finding the G. C. M. under like circumstances.

Find the L. C. M. of 1247 and 1769.

```
1247)1769(1
     1247
   2) 522
   9) 261
     29)1247(43
         116
          87
          87
```

Hence, the G. C. M. $= 29$; and $1247 = 29 \times 43$,
also $1769 = 29 \times 61$.
$\therefore$ the L. C. M. $= 29 \times 43 \times 61 = 1247 \times 61 = 76{,}067$.

243. From this process it will be seen that:

The L. C. M. *of two numbers may be found by dividing one of the numbers by their* G. C. M. *and multiplying the quotient by the other number.*

The L. C. M. of two prime numbers, or of two numbers prime to each other, is their product.

EXERCISE XVII.

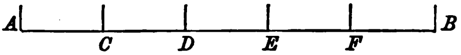
Find the L. C. M. of

1. 424 and 583.
2. 319 and 407.
3. 1,679 and 1,932.
4. 1,003 and 2,419.
5. 1,003 and 1,357.
6. 899 and 961.
7. 407, 703, 444.
8. 411, 959, 2,055.
9. 221 and 351.
10. 1,426 and 989.
11. 3,864, 3,404, 3,657.
12. 539 and 253.
13. 2,943, 2,616, 4,578.
14. 2,863 and 1,151.
15. 1,177, 1,391, 1,819.
16. 5,317 and 2,863.
17. 12,703 and 12,879.
18. 23,309 and 10,753.
19. 4,939 and 3,143.
20. 4,199 and 6,137.

CHAPTER XI.

COMMON FRACTIONS.

244. Instead of dividing the unit into 10 equal parts, 100 equal parts, and so on, it is often more convenient to divide it into halves, thirds, quarters, or other equal parts. These are called **common fractions.**



245. If a line AB be divided into 5 equal parts, at the points C, D, E, and F, the parts AC, AD, AE, and AF are fractions of the whole, being respectively *one-fifth*, *two-fifths*, *three-fifths*, *four-fifths* of the whole line, and are written $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.

246. The expression $\frac{4}{5}$ means:

I. 4 of the parts when 1 unit has been divided into 5 equal parts.

II. $\frac{1}{5}$ of 4 units; for if *four* units be divided into 5 equal parts, one of these parts will be *four* times as great as one of the parts obtained by dividing *one* unit into 5 equal parts; that is, will be equal to $\frac{4}{5}$ of one unit.

III. The quotient of 4 divided by 5.

247. In the fraction $\frac{4}{5}$, the lower figure shows into how many equal parts the whole has been divided, and is therefore a **divisor.**

But since it shows the number of parts into which the whole has been divided, it shows the **name** of each part, and is therefore called the **denominator.**

The upper figure shows how many of these parts are taken, and is therefore called the **numerator.***

248. It will be observed that a figure written *above* the line serves a very different purpose from that of a figure written *below* the line. A figure written above the line denotes **number,** a figure below the line, **name.**

249. The expressions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{1}{6}$, $\frac{4}{7}$, $\frac{17}{21}$, are read one-half, two-thirds, three-fourths, three-fifths, one-sixth, four-sevenths, seventeen twenty-firsts.

In like manner, all other fractions are read by reading first the numerator, and then the denominator as an ordinal, with an *s* added if the numerator is more than one.

Read:

$\frac{5}{13}$, $\frac{7}{10}$, $\frac{18}{37}$, $\frac{99}{1000}$, $\frac{8}{21}$, $\frac{34}{89}$, $\frac{13}{32}$, $\frac{55}{144}$, $\frac{55}{56}$, $\frac{17}{55}$, $\frac{21}{23}$, $\frac{19}{33}$, $\frac{7}{12}$, $\frac{5}{11}$.

250. The numerator and denominator are called the **terms** of a fraction.

251. A **proper fraction** is one of which the numerator is less than the denominator; as $\frac{5}{7}$.

252. An **improper fraction** is one of which the numerator equals or exceeds the denominator; as $\frac{7}{7}$, $\frac{15}{2}$.

It is obvious that when the numerator is greater than the denominator, more than one unit must be regarded as divided into parts; thus, $\frac{29}{13}$ means that three units have been divided each into thirteen equal parts, and that all the parts of two units and three parts of the third are taken.

253. A **mixed number** is an expression consisting of a whole number and a fraction; as $5\frac{4}{7}$.

It means that some entire units are taken, and a fraction of another unit.

---

* Numerator and denominator are derived from the Latin *numerare,* to count, and *denominare,* to name.

254. An improper fraction represents a quantity which can also be represented by a whole number, or else by a mixed number. Thus, $\frac{17}{7} = 2\frac{3}{7}$.

For, if we suppose several units to be divided each into seven equal parts, and we take 17 of these parts, 14 (that is, 2 × 7) will make two units, and the three parts remaining will be three-sevenths of another unit.

255. To reduce an improper fraction to a whole or mixed number,

Divide the numerator by the denominator.

The quotient will be the whole number, and the remainder, if there be any, will be the numerator of the fractional part, of which the denominator is the same as the denominator of the improper fraction.

EXERCISE XVIII.

Reduce to whole or mixed numbers:

1. $\frac{13}{7}$.
2. $\frac{21}{8}$.
3. $\frac{25}{4}$.
4. $\frac{107}{11}$.
5. $\frac{213}{16}$.
6. $\frac{242}{22}$.
7. $\frac{374}{25}$.
8. $\frac{481}{18}$.
9. $\frac{3823}{75}$.
10. $\frac{4684}{177}$.
11. $\frac{2829}{753}$.
12. $\frac{299}{23}$.
13. $\frac{529}{29}$.
14. $\frac{786}{42}$.
15. $\frac{8975}{25}$.

256. A whole number, or a mixed number, represents a quantity which can also be represented by an improper fraction. Thus, $5\frac{7}{13} = \frac{72}{13}$.

For each unit contains 13 *thirteenths;* therefore 5 units contain 5 × 13, or 65, *thirteenths;* which, together with the 7 *thirteenths,* make 72 *thirteenths.* Therefore,

257. To reduce a mixed number to an improper fraction,

Multiply the whole number by the denominator of the fraction, and to the product add the numerator; under this sum write the denominator.

258. A whole number may be expressed as a fraction with any given denominator. Thus, $7 = \frac{63}{9}$.

For, as each unit contains 9 *ninths*, 7 units contain 7 × 9 *ninths*, that is, 63 *ninths*.

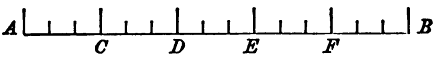
A whole number may be written in a fractional form with 1 for a denominator. Thus, $4 = \frac{4}{1}$.

EXERCISE XIX.

Reduce to improper fractions:

1. $3\frac{4}{9}$.
2. $5\frac{9}{10}$.
3. $12\frac{4}{11}$.
4. $10\frac{7}{19}$.
5. $8\frac{3}{7}$.
6. $12\frac{13}{15}$.
7. $84\frac{17}{20}$.
8. $864\frac{13}{97}$.
9. $41\frac{83}{1000}$.
10. $41\frac{3}{10000}$.
11. $400\frac{127}{400}$.
12. $5000\frac{83}{5000}$.
13. $10{,}000\frac{14}{15}$.
14. $3001\frac{83}{700}$.
15. $73\frac{72}{99}$.
16. Express 8, 7, 3, 5, 12, 13, 18, 20, 25 in the form of fractions, each having 5 for a denominator.
17. Express 21 in the form of fractions, having for denominators 3, 5, 7, 8, 12, 13, 20, 25, 30, 37.
18. Express 12, 15, 23 in the form of fractions each having for denominators 12, 15, 23, respectively.

259. If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the **value** of the fraction is not altered.



Thus, if the line AB be divided into 5 equal parts at the points C, D, E, and F, then AF is $\frac{4}{5}$ of AB. (1)

Now, if each of the parts be subdivided into 3 equal parts, AB will contain 15 of these subdivisions, and AF 12 of these subdivisions.

Therefore, AF is $\frac{12}{15}$ of AB. (2)

Comparing (1) and (2), $\frac{12}{15} = \frac{4}{5}$. Therefore,

260. To reduce a fraction to lower terms,

Divide the numerator and denominator by any common factor.

A fraction is expressed in its lowest terms when both the numerator and denominator are divided by the greatest common divisor.

(1) Reduce $\frac{324}{756}$ to its lowest terms.

$$\frac{324}{756}=\frac{36}{84}=\frac{9}{21}=\frac{3}{7};$$

the common divisors used being 9, 4, and 3.

(2) Reduce $\frac{1261}{1649}$ to its lowest terms.

Since no common factor can be readily detected, we find the G. C. M. thus:

$$\begin{array}{rrl} 1261\,) & 1649\,(& 1 \\ & \underline{1261} & \\ 4\,\rfloor & 388 & \\ 97\,) & 1261\,(& 13 \\ & \underline{1261} & \end{array}$$

Divide 1261 and 1649 each by 97, their G. C. M.

Then, $\frac{1261}{1649}=\frac{13}{17}$.

EXERCISE XX.

Reduce to lowest terms:

1. $\frac{120}{192}$.
2. $\frac{105}{135}$.
3. $\frac{928}{1320}$.
4. $\frac{1728}{2448}$.
5. $\frac{1296}{6561}$.
6. $\frac{2310}{3080}$.
7. $\frac{1848}{2352}$.
8. $\frac{3960}{12672}$.
9. $\frac{1848}{3003}$.
10. $\frac{924}{1092}$.
11. $\frac{2640}{2970}$.
12. $\frac{324}{1092}$.
13. $\frac{6732}{9108}$.
14. $\frac{6840}{27360}$.
15. $\frac{5760}{7000}$.
16. $\frac{875}{10000}$.
17. $\frac{2208}{4140}$.
18. $\frac{1015}{1566}$.
19. $\frac{516}{2107}$.
20. $\frac{3872}{92807}$.
21. $\frac{78473}{94653}$.
22. $\frac{17596}{26145}$.
23. $\frac{44828}{61087}$.
24. $\frac{339}{1248}$.
25. $\frac{1177}{2675}$.
26. $\frac{11445}{15369}$.
27. $\frac{14141}{16289}$.
28. $\frac{881496}{104763}$.
29. $\frac{65065}{999999}$.
30. $\frac{428571}{999999}$.

MULTIPLICATION OF FRACTIONS.

261. 4×2 *dollars* $=8$ *dollars.*

4×2 *sevenths* $=8$ *sevenths.*

If two like quantities are taken 4 times, the result will be 4 times 2 of the same quantities.

$\frac{2}{7}\times 14$ means $\frac{2}{7}$ of 14, which equals 4. Therefore,

262. To find the product of a whole number and a fraction,

Find the product of the numerator and whole number, and divide the result by the denominator.

A factor common to the whole number and the denominator of the fraction may be cancelled. For, cancelling a factor common to the whole number and the denominator of the fraction *before* the multiplication, is evidently equivalent to dividing the numerator and denominator of the resulting fraction by that factor *after* the multiplication. Which may be done by §§ 259, 260.

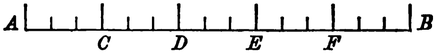
EXERCISE XXI.

Find the product of:

1. $\frac{3}{4} \times 2$.
2. $\frac{3}{4} \times 9$.
3. $10 \times \frac{2}{5}$.
4. $15 \times \frac{3}{5}$.
5. $\frac{9}{21} \times 7$.
6. $16 \times \frac{5}{8}$.
7. $\frac{5}{8} \times 2$.
8. $\frac{2}{15} \times 5$.
9. $27 \times \frac{5}{9}$.
10. $\frac{13}{20} \times 2$.
11. $\frac{13}{20} \times 3$.
12. $\frac{13}{20} \times 4$.
13. $5 \times \frac{13}{20}$.
14. $6 \times \frac{13}{20}$.
15. $7 \times \frac{13}{20}$.
16. $8 \times \frac{13}{20}$.
17. $\frac{13}{20} \times 10$.
18. $\frac{13}{20} \times 12$.
19. $\frac{13}{20} \times 15$.
20. $\frac{13}{20} \times 20$.

263. To find the value of a fraction of a fraction; or,
To multiply a fraction by a fraction:

Multiply $\frac{4}{5}$ by $\frac{2}{3}$.



$\frac{4}{5}$ multiplied by $\frac{2}{3}$ means $\frac{2}{3}$ of $\frac{4}{5}$.

If the line AB be divided into 5 equal parts at the points C, D, E, and F, AF will be $\frac{4}{5}$ of AB.

Now, if each part be subdivided into three equal parts, there will evidently be 15 such parts in the whole line, and each part will be $\frac{1}{15}$ of the line.

That is, $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$ of the whole.

$\frac{1}{3}$ of $\frac{4}{5}$ will be $\frac{1}{15} + \frac{1}{15} + \frac{1}{15} + \frac{1}{15}$, or $\frac{4}{15}$ of the whole.

And $\frac{2}{3}$ of $\frac{4}{5}$ will be *twice* $\frac{4}{15}$, that is, $\frac{8}{15}$ of the whole.

Therefore,

264. To multiply a fraction by a fraction,

Find the product of the numerators for the numerator of the product, and of the denominators for the denominator of the product.

Mixed numbers must first be reduced to improper fractions.

Any factor common to a numerator and denominator should be cancelled before the multiplication.

$$(1)\quad \tfrac{12}{25} \times 2\tfrac{13}{16} \times \tfrac{10}{21} = \tfrac{12}{25} \times \tfrac{45}{16} \times \tfrac{10}{21},$$

$$= \frac{\overset{\cancel{3}}{\cancel{12}}}{\underset{\cancel{5}}{\cancel{25}}} \times \frac{\overset{9}{\cancel{45}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{16}}} \times \frac{\overset{\cancel{2}}{\cancel{10}}}{\underset{7}{\cancel{21}}} = \frac{9}{14}.$$

12 and 16 contain a common factor, 4, which is cancelled;

3 (the quotient of 12 by 4) and 21 contain a common factor, 3;

45 and 25 the common factor 5;

5 and 10 the common factor 5;

finally, 2 and 4, the common factor 2; and the common factors are all cancelled.

There remains in the numerator 9; and in the denominator 2 × 7, from which is obtained the simple fraction $\frac{9}{14}$.

Exercise XXII.

Simplify:

1. $\frac{2}{3}$ of $\frac{7}{11}$.
2. $\frac{3}{7}$ of $2\frac{1}{10}$.
3. $\frac{3}{7}$ of $\frac{5}{9}$.
4. $2\frac{2}{5} \times 2\frac{1}{2}$.
5. $4\frac{4}{5} \times 2\frac{1}{7}$.
6. $4\frac{5}{6} \times 9\frac{1}{3}$.
7. $\frac{1}{3}$ of $\frac{3}{5}$ of 10.
8. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{2}{3}$.
9. $\frac{4}{5} \times \frac{5}{6} \times \frac{3}{7}$ of $4\frac{1}{5}$.
10. $\frac{5}{6}$ of $4\frac{1}{2}$.
11. $\frac{8}{9}$ of $\frac{9}{10}$ of $\frac{5}{7}$ of $\frac{3}{4}$ of $\frac{1}{5}$ of $15\frac{3}{4}$.
12. $5\frac{3}{4} \times 8\frac{2}{5}$.
13. $\frac{2}{3} \times \frac{4}{7} \times \frac{7}{15} \times 7\frac{1}{2}$.
14. $\frac{3}{5}$ of $\frac{10}{27}$ of $\frac{9}{20}$ of $8\frac{1}{3}$.
15. $\frac{8}{11} \times \frac{20}{21} \times \frac{35}{48} \times 2\frac{10}{19}$.
16. $\frac{42}{43} \times \frac{13}{105} \times 1\frac{7}{308}$.
17. $\frac{5}{8} \times \frac{120}{121} \times \frac{66}{85} \times 17$.
18. $\frac{38}{39} \times \frac{52}{57} \times \frac{69}{86} \times 1\frac{20}{23}$.
19. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 10.

20. $\frac{7}{25}$ of $\frac{8}{11}$ of 30.

21. $\frac{113}{355} \times \frac{85}{226} \times \frac{12}{35} \times 1\frac{3}{4}$.

22. $\frac{7}{8} \times \frac{3}{4} \times \frac{8}{21} \times \frac{4}{5}$ of $\frac{5}{8}$ of $\frac{3}{4}$ of 8.

23. $\frac{2}{13}$ of $\frac{39}{40}$ of $\frac{52}{117}$.

24. $\frac{9}{11} \times \frac{7}{12} \times \frac{22}{63} \times 48$.

25. $\frac{39}{40}$ of $\frac{7}{65}$ of $\frac{25}{42}$ of 12.

26. $1\frac{2}{3}$ of $4\frac{1}{2}$ of $\frac{3}{5}$.

27. $2\frac{4}{9} \times 1\frac{3}{7} \times 1\frac{13}{15} \times 8$.

28. $3\frac{5}{7}$ of $2\frac{1}{2}$ of $1\frac{8}{13}$ of $1\frac{4}{11}$.

29. $\frac{11}{12} \times 5\frac{1}{3} \times 4\frac{1}{8} \times \frac{7}{22} \times 5$.

30. $\frac{2}{5}$ of $\frac{7}{15} \times 8\frac{2}{7} \times \frac{6}{29}$ of $1\frac{17}{18}$.

31. $\frac{17}{32} \times \frac{27}{38} \times \frac{128}{153}$.

32. $\frac{292}{465} \times \frac{573}{657} \times \frac{248}{1719}$.

33. $\frac{1205}{2763}$ of $\frac{712}{2169}$ of $\frac{1585}{1068}$.

Division of Fractions.

265. The reciprocal of a number is 1 divided by the number; thus, the reciprocal of 4 is $\frac{1}{4}$, for $4 \times \frac{1}{4} = 1$.

The reciprocal of a fraction is the fraction with its terms interchanged; thus, $\frac{7}{8}$ is the reciprocal of $\frac{8}{7}$, for $\frac{7}{8} \times \frac{8}{7} = 1$.

266. Multiplying by the reciprocal of a number is the same as dividing by the number. See § 162. Therefore,

To divide by a whole number or a fraction,

Multiply by its reciprocal.

Thus:

$$\frac{3}{5} \div 4 = \frac{1}{4} \text{ of } \frac{3}{5} = \frac{1}{4} \times \frac{3}{5} = \frac{3}{20};$$
$$\frac{3}{5} \div \frac{7}{8} = \frac{8}{7} \text{ of } \frac{3}{5} = \frac{8}{7} \times \frac{3}{5} = \frac{24}{35}.$$

Mixed numbers must first be reduced to improper fractions,

Exercise XXIII.

Divide:

1. $\frac{24}{35}$ by 6.

2. $\frac{10}{11}$ by 5.

3. $\frac{3}{7}$ by 8.

4. $18\frac{2}{3}$ by 7.

5. $\frac{5}{8}$ by $\frac{3}{4}$.

6. $\frac{12}{13}$ by $\frac{3}{8}$.

7. $1\frac{3}{4}$ by $3\frac{1}{3}$.

8. $5\frac{1}{5}$ by $4\frac{2}{3}$.

9. $8\frac{2}{9}$ by $4\frac{1}{9}$.

10. $7\frac{1}{5}$ by $4\frac{2}{7}$.

11. $6\frac{3}{4}$ by $9\frac{1}{2}$.

12. $8\frac{2}{3}$ by $4\frac{2}{3}$.

13. $3\frac{8}{9}$ by $\frac{14}{27}$.

14. $4\frac{3}{7}$ by $6\frac{8}{9}$.

15. 5 by $4\frac{2}{7}$.

16. $3\frac{4}{5}$ of $2\frac{1}{2}$ by $1\frac{1}{2}$ of $2\frac{1}{9}$.

17. $2\frac{2}{7}$ by $3\frac{1}{3}$ of $1\frac{1}{15}$.

18. $2\frac{9}{11}$ of $5\frac{1}{2}$ by $7\frac{3}{4}$.

19. $5\frac{3}{5}$ of $8\frac{1}{3}$ of $1\frac{4}{7}$ by $2\frac{1}{10}$ of $5\frac{5}{9}$.

LEAST COMMON DENOMINATOR.

267. To reduce fractions to equivalent fractions having the least common denominator:

Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to equivalent fractions having the least common denominator.

The L. C. M. of 3, 4, 6 = 12.

If both terms of $\frac{2}{3}$ be multiplied by 4, the value of the fraction will not be altered, but the form will be changed to $\frac{8}{12}$.

If both terms of $\frac{3}{4}$ be multiplied by 3, the equivalent fraction will be $\frac{9}{12}$.

And if both terms of $\frac{5}{6}$ be multiplied by 2, the equivalent fraction will be $\frac{10}{12}$.

The multipliers, 4, 3, and 2, are obtained by dividing the L. C. M. of the denominators, 12, by the respective denominators of the given fractions. Therefore,

268. To reduce fractions to equivalent fractions having the least common denominator:

Find the L. C. M. *of the denominators.*

Divide the L. C. M. *by the denominator of each fraction.*

Multiply the first numerator by the first quotient, the second numerator by the second quotient, and so on.

The products will be the numerators of the equivalent fractions.

The L. C. M. *of the given denominators will be the denominator of each of the equivalent fractions.*

Reduce $\frac{3}{4}$, $\frac{7}{8}$, $\frac{11}{12}$ to equivalent fractions having the least common denominator (L. C. D.).

$$4 = 2^2,$$
$$8 = 2^3,$$
$$12 = 2^2 \times 3.$$

Hence, the L. C. D. $= 2^3 \times 3$, or 24.

$$\frac{3}{4}=\frac{18}{24},$$
$$\frac{7}{8}=\frac{21}{24},$$
$$\text{and } \frac{11}{12}=\frac{22}{24}.$$

The result may be written as follows:

$$\frac{18 \quad 21 \quad 22}{24}$$

By this operation the parts represented by the given fractions have been subdivided into smaller parts all of *one size*, and the numerators of the resulting fractions show the *number* of these smaller parts contained in the given fractions. Thus, the quantities denoted by $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{11}{12}$ are each subdivided into 24ths of the unit, and contain respectively 18, 21, and 22 of these subdivisions.

269. *Fractions may be compared by first reducing them to equivalent fractions having a common denominator.*

Determine the greater of the fractions $\frac{5}{7}$ and $\frac{11}{16}$.

In this case the least common denominator is 112.

$$\text{Hence, } \frac{5}{7}=\frac{80}{112},$$
$$\frac{11}{16}=\frac{77}{112}.$$
$$\therefore \frac{5}{7} \text{ is greater than } \frac{11}{16}.$$

EXERCISE XXIV.

Express with least common denominator:

1. $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$.
2. $\frac{2}{3}, \frac{5}{9}, \frac{7}{8}, \frac{9}{10}$.
3. $\frac{5}{6}, \frac{1}{8}, \frac{5}{21}, \frac{19}{35}$.
4. $\frac{2}{15}, \frac{7}{20}, \frac{3}{25}, \frac{8}{45}$.
5. $\frac{12}{25}, \frac{17}{40}, \frac{13}{60}, \frac{19}{75}$.
6. $\frac{3}{8}, \frac{7}{30}, \frac{4}{35}, \frac{3}{28}, \frac{19}{24}$.
7. $\frac{11}{16}, \frac{7}{18}, \frac{13}{20}, \frac{23}{30}, \frac{17}{54}$.
8. Which is the greater, $\frac{13}{20}$ or $\frac{17}{25}$? $\frac{5}{6}$ or $\frac{7}{9}$? $\frac{3}{5}$ or $\frac{7}{12}$?
9. Arrange the fractions $\frac{7}{12}, \frac{11}{18}, \frac{13}{24}$ in order of magnitude.
10. Arrange the fractions $\frac{5}{12}, \frac{8}{15}, \frac{4}{11}, \frac{7}{18}$ in order of magnitude.

Addition of Fractions.

Find the sum of $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{9}$, $\frac{5}{12}$.

$$\begin{aligned} 4 &= 2^2, \\ 6 &= 2 \times 3, \\ 9 &= 3^2, \\ 12 &= 2^2 \times 3. \end{aligned}$$

Hence, the L. C. D. $= 2^2 \times 3^2 = 36$.

$$\begin{aligned} \text{And } \tfrac{3}{4} + \tfrac{5}{6} + \tfrac{7}{9} + \tfrac{5}{12} &= \tfrac{27+30+28+15}{36}, \\ &= \tfrac{100}{36} = 2\tfrac{28}{36} = 2\tfrac{7}{9}. \textit{ Ans.} \end{aligned}$$

270. Therefore, to add fractions:

Reduce the fractions to equivalent fractions having the least common denominator.

Add the numerators of the equivalent fractions.

Write the result over the least common denominator.

Find the sum of $4\frac{11}{20}$, $2\frac{7}{15}$, $5\frac{5}{12}$.

$$\begin{aligned} 20 &= 2^2 \times 5, \\ 15 &= 3 \times 5, \\ 12 &= 2^2 \times 3. \end{aligned}$$

Hence, the L. C. D. $= 2^2 \times 3 \times 5 = 60$.

$$\begin{aligned} \text{And } 4\tfrac{11}{20} + 2\tfrac{7}{15} + 5\tfrac{5}{12} &= 11\tfrac{33+28+25}{60}, \\ &= 11\tfrac{86}{60}, \\ &= 12\tfrac{26}{60}, \\ &= 12\tfrac{13}{30}. \end{aligned}$$

If any of the terms are integers, or mixed numbers, add together *separately* the integers and the fractions, and find the sum of the results.

EXERCISE XXV.

Find the sum of:

1. $\frac{1}{2}+\frac{3}{2}$.
2. $\frac{1}{3}+\frac{2}{3}+\frac{1}{3}$.
3. $\frac{1}{4}+\frac{1}{4}+\frac{3}{4}$.
4. $1\frac{1}{2}+2\frac{1}{2}$.
5. $1\frac{1}{3}+2\frac{2}{3}$.
6. $3\frac{1}{4}+\frac{3}{4}$.
7. $2\frac{3}{5}+3\frac{4}{5}$.
8. $1\frac{7}{8}+\frac{3}{8}$.
9. $\frac{9}{17}+\frac{3}{17}+\frac{14}{17}+\frac{11}{17}$.
10. $8\frac{9}{17}+6\frac{3}{17}+5\frac{14}{17}+\frac{11}{17}$.
11. $\frac{4}{5}+\frac{5}{6}$.
12. $\frac{3}{4}+\frac{7}{8}$.
13. $\frac{1}{2}+\frac{1}{6}$.
14. $\frac{4}{15}+\frac{11}{20}$.
15. $\frac{5}{16}+\frac{11}{24}$.
16. $12\frac{5}{8}+7\frac{3}{16}$.
17. $85\frac{7}{12}+27\frac{11}{18}$.
18. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$.
19. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$.
20. $\frac{5}{6}+\frac{11}{12}+\frac{8}{15}+\frac{7}{20}+\frac{13}{30}$.
21. $5\frac{17}{20}+11\frac{19}{30}+24\frac{21}{40}+\frac{9}{50}+17\frac{8}{15}+14+11\frac{5}{12}$.
22. $9\frac{4}{7}+15\frac{11}{28}+163\frac{17}{63}+1\frac{11}{42}+10\frac{1}{4}$.
23. $3\frac{3}{5}+4\frac{2}{3}+1\frac{5}{6}+2$.
24. $1\frac{3}{20}+2\frac{2}{25}+5\frac{7}{30}+\frac{4}{15}$.
25. $\frac{2}{7}+1\frac{4}{9}+2+3\frac{3}{8}+4\frac{5}{12}$.
26. $4\frac{4}{9}+3\frac{3}{8}+2\frac{2}{7}+1\frac{1}{6}+\frac{9}{14}$.
27. $\frac{11}{35}+\frac{7}{40}+10+\frac{23}{60}$.
28. $\frac{27}{50}+\frac{29}{60}+\frac{31}{80}+\frac{33}{100}+\frac{37}{240}$.
29. $2+\frac{2}{3}+1\frac{3}{4}+4\frac{8}{9}+5\frac{13}{24}$.
30. $3\frac{5}{8}+6+\frac{4}{11}+2\frac{3}{10}+5\frac{5}{16}+\frac{9}{20}$.
31. $\frac{8}{15}+\frac{7}{18}+3\frac{17}{20}+1\frac{19}{24}+2\frac{19}{120}$.
32. $\frac{5}{14}+\frac{6}{11}+9\frac{1}{2}$.
33. $20\frac{5}{12}+11\frac{7}{20}+5\frac{1}{8}+305$.
34. $\frac{11}{38}+\frac{14}{57}+\frac{17}{76}$.
35. $\frac{5}{17}+\frac{11}{34}+\frac{14}{51}+\frac{19}{68}$.
36. $317\frac{2}{5}+17\frac{8}{51}+4\frac{9}{10}+\frac{7}{15}+6\frac{2}{3}+\frac{5}{17}$.
37. $4\frac{7}{15}+8\frac{5}{21}+4\frac{7}{11}+5\frac{2}{7}+5\frac{4}{5}+\frac{2}{3}$.
38. $3\frac{2}{3}+5\frac{3}{40}+8\frac{7}{240}+\frac{39}{60}+1\frac{29}{2880}$.
39. $4\frac{5}{13}+7\frac{5}{39}+5\frac{47}{78}+275\frac{37}{156}+2\frac{57}{91}$.
40. $\frac{17}{88}+7\frac{5}{12}+6\frac{3}{44}+400\frac{3}{28}+51\frac{25}{56}$.

SUBTRACTION OF FRACTIONS.

(1) From $\frac{17}{24}$ take $\frac{5}{18}$.

$$24 = 2^3 \times 3,$$
$$18 = 2 \times 3^2.$$

Hence, the L. C. D. $= 2^3 \times 3^2 = 72.$

$$\tfrac{17}{24} - \tfrac{5}{18} = \tfrac{51-20}{72} = \tfrac{31}{72}. \text{ Ans.}$$

271. Therefore, to subtract one fraction from another:

Reduce the fractions to equivalent fractions having the least common denominator.

Subtract the numerator of the subtrahend from the numerator of the minuend.

Write the result over the least common denominator.

(2) Subtract $3\frac{11}{24}$ from $4\frac{11}{15}$.

$$24 = 2^3 \times 3,$$
$$15 = 3 \times 5.$$

Hence, the L. C. D. $= 2^3 \times 3 \times 5 = 120.$

$$4\tfrac{11}{15} - 3\tfrac{11}{24} = 1\tfrac{88-55}{120} = 1\tfrac{33}{120} = 1\tfrac{11}{40}. \text{ Ans.}$$

If the terms are mixed numbers, subtract *separately* the integers and the fractions, and unite the results.

(3) Subtract $2\frac{7}{8}$ from $5\frac{5}{12}$.

$$5\tfrac{5}{12} - 2\tfrac{7}{8} = 3\tfrac{10-21}{24} = 2\tfrac{34-21}{24} = 2\tfrac{13}{24}. \text{ Ans.}$$

The difference between $5\frac{5}{12}$ and $2\frac{7}{8}$ is $3\frac{10-21}{24}$.

Since $\frac{21}{24}$ cannot be subtracted from $\frac{10}{24}$, 1 is taken from 3 and added to $\frac{10}{24}$, making $\frac{34}{24}$.

(4) From 8 take $2\frac{31}{34}$.

$$8 = 7\tfrac{34}{34},$$
$$7\tfrac{34}{34} - 2\tfrac{31}{34} = 5\tfrac{34-31}{34} = 5\tfrac{3}{34}. \text{ Ans.}$$

EXERCISE XXVI.

Find the value of:

1. $52\frac{1}{8} - 46$.
2. $\frac{5}{9} - \frac{2}{9}$.
3. $\frac{3}{4} - \frac{2}{3}$.
4. $\frac{8}{15} - \frac{5}{12}$.
5. $\frac{11}{18} - \frac{3}{14}$.
6. $4 - \frac{1}{2}$.
7. $7 - \frac{2}{3}$.
8. $3 - \frac{5}{6}$.
9. $8 - \frac{3}{7}$.
10. $5 - \frac{4}{5}$.
11. $5 - \frac{7}{9}$.
12. $6\frac{1}{3} - 5\frac{1}{6}$.
13. $4\frac{2}{5} - 3\frac{6}{7}$.
14. $7\frac{1}{3} - 2\frac{3}{10}$.
15. $7\frac{2}{5} - 4\frac{3}{5}$.
16. $6\frac{2}{3} - 2\frac{3}{4}$.
17. $9\frac{4}{5} - 4\frac{5}{6}$.
18. $4\frac{2}{3} - \frac{1}{2}$.
19. $6\frac{3}{4} - 4\frac{2}{3}$.
20. $7\frac{1}{2} - 2\frac{3}{4}$.
21. $8\frac{1}{5} - 4\frac{4}{7}$.
22. $85\frac{7}{22} - 27\frac{11}{16}$.
23. $8\frac{7}{10} - 2\frac{11}{16}$.
24. $10 - 3\frac{5}{8}$.
25. $120\frac{21}{32} - 110\frac{13}{24}$.
26. $5\frac{17}{28} - \frac{27}{35}$.
27. $13\frac{3}{40} - 2\frac{15}{44}$.
28. $2\frac{151}{240} - 1\frac{163}{192}$.
29. $4 - 1\frac{2817}{4000}$.
30. $1473 - 279\frac{11}{12}$.
31. $1473\frac{5}{18} - 279\frac{11}{12}$.
32. $1473\frac{7}{18} - 279\frac{11}{12}$.
33. $278\frac{15}{16} - 30\frac{5}{12}$.
34. $125\frac{5}{22} - 10\frac{17}{33}$.
35. $118\frac{5}{11} - 17\frac{3}{14}$.
36. $94\frac{5}{11} - 91\frac{13}{14}$.
37. $7\frac{5}{21} - 2\frac{11}{14}$.
38. $\frac{235}{357} - \frac{13}{51}$.
39. $\frac{17}{68} - \frac{29}{108}$.
40. $\frac{9}{38} - \frac{48}{209}$.
41. $\frac{146}{273} - \frac{268}{637}$.
42. $\frac{359}{360} - \frac{199}{200}$.

PLUS AND MINUS TERMS.

Simplify $5\frac{4}{5} - 4\frac{3}{4} + 3\frac{2}{3} - 2\frac{7}{10}$.

$$5\frac{4}{5} + 3\frac{2}{3} = 8\frac{12+10}{15} = 8\frac{22}{15} = 9\frac{7}{15},$$
$$4\frac{3}{4} + 2\frac{7}{10} = 6\frac{15+14}{20} = 6\frac{29}{20} = 7\frac{9}{20},$$
$$\text{and } 9\frac{7}{15} - 7\frac{9}{20} = 2\frac{28-27}{60} = 2\frac{1}{60}.$$

272. Hence, to simplify an expression consisting of plus and minus terms,

Subtract the sum of the minus terms from the sum of the plus terms.

COMPLEX FRACTIONS.

273. A complex fraction is one which has a fraction in the numerator or in the denominator, or in both.

274. The simplest meaning to give to a complex fraction is that of an indicated division. Hence,

To simplify a complex fraction,

Divide the numerator by the denominator.

$$(1)\quad \frac{3\frac{4}{5}}{20}=\frac{19}{5}\div 20=\frac{19}{5}\times\frac{1}{20}=\frac{19}{100}.$$

$$(2)\quad \frac{2\frac{5}{6}}{4\frac{6}{7}}=\frac{17}{6}\div\frac{34}{7}=\frac{\cancel{17}}{6}\times\frac{7}{\underset{2}{\cancel{34}}}=\frac{7}{12}.$$

$$(3)\quad \frac{2\frac{2}{3}-1\frac{5}{9}}{1\frac{5}{6}-1\frac{1}{8}}.$$

$$2\frac{2}{3}-1\frac{5}{9}=1\frac{6-5}{9}=1\frac{1}{9};\quad 1\frac{5}{6}-1\frac{1}{8}=\frac{20-3}{24}=\frac{17}{24};$$

$$1\frac{1}{9}\div\frac{17}{24}=\frac{10}{\underset{3}{\cancel{9}}}\times\frac{\overset{8}{\cancel{24}}}{17}=\frac{80}{51}=1\frac{29}{51}.\quad \textit{Ans.}$$

A complex fraction may be simplified by multiplying both its terms by the smallest number that will make them integral. This multiplier will always be the L. C. M. of the denominators of the fractions contained in the terms of the given fraction.

$$(4)\quad \frac{4\frac{7}{9}-3\frac{1}{2}-2\frac{1}{3}+1\frac{7}{18}}{3\frac{5}{9}-2\frac{2}{3}+2\frac{1}{2}-\frac{7}{18}}=\frac{86-63-42+25}{64-48+45-7}=\frac{6}{54}=\frac{1}{9}.$$

Here each term of the numerator and denominator is made integral by multiplying it by 18, the L. C. M. of the denominators of the fractions contained in the numerator and denominator of the given fraction.

$$(5)\quad \frac{6\frac{3}{4}-1\frac{8}{9}}{\frac{7}{12}\text{ of }1\frac{2}{3}}=\frac{6\frac{3}{4}-1\frac{8}{9}}{\frac{35}{36}}=\frac{243-68}{35}=\frac{175}{35}=5.$$

Here the compound term $\frac{7}{12}$ of $1\frac{2}{3}$ is first reduced to the simple term $\frac{35}{36}$, and then the numerator and denominator of the resulting fraction is multiplied by 36, the L. C. M. of the small fractions.

Compound terms must first be reduced to simple terms.

EXERCISE XXVII.

Simplify:

1. $\dfrac{2\frac{3}{11}}{3\frac{3}{4}}$.

2. $\dfrac{3}{7\frac{1}{5}}$.

3. $\dfrac{17\frac{1}{7}}{13\frac{1}{3}}$.

4. $\dfrac{\frac{5}{6}}{8\frac{1}{3}}$.

5. $\dfrac{5\frac{1}{6}}{8\frac{4}{11}}$.

6. $\dfrac{1\frac{4}{5}\text{ of }3\frac{1}{7}}{4\frac{1}{3}\text{ of }\frac{9}{10}}$.

7. $\dfrac{2\frac{1}{2}-1\frac{5}{6}}{1\frac{5}{6}-1\frac{5}{8}}$.

8. $\dfrac{10\frac{2}{5}-1\frac{5}{7}}{7\frac{1}{8}-3\frac{3}{40}}$.

9. $\dfrac{\frac{3}{7}\text{ of }2\frac{1}{17}}{1\frac{2}{3}+2\frac{3}{7}}$.

10. $\dfrac{6\frac{3}{4}-1\frac{5}{14}}{2\frac{1}{6}+1\frac{3}{7}}$.

11. $\dfrac{5\frac{4}{5}+2\frac{3}{7}}{4\frac{2}{3}-3\frac{11}{15}}$.

12. $\dfrac{8\frac{3}{4}}{14}-\dfrac{\frac{3}{5}}{1\frac{1}{7}}$.

13. $\dfrac{3\frac{3}{7}}{11\frac{1}{4}}\text{ of }\dfrac{3\frac{3}{8}}{2\frac{2}{5}}$.

14. $\dfrac{5\frac{3}{8}-4\frac{11}{12}}{5\frac{3}{8}-2\frac{17}{18}}$.

15. $\dfrac{2\frac{3}{4}+2\frac{7}{8}}{4\frac{3}{4}-3\frac{1}{7}}$.

16. $\dfrac{2\frac{3}{5}\times\frac{9}{11}}{3\frac{5}{7}+4\frac{1}{8}}$.

17. $\dfrac{\frac{17}{20}+\frac{11}{15}+\frac{7}{10}+\frac{4}{5}}{\frac{17}{20}-\frac{11}{15}+\frac{7}{10}-\frac{4}{5}}$.

18. $\dfrac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{2}-2\frac{1}{7}}$.

19. $\dfrac{2\frac{27}{40}-4\frac{4}{7}+3\frac{1}{8}}{5\frac{6}{7}-4\frac{7}{8}+\frac{3}{5}}$.

20. $\dfrac{1\frac{1}{4}\times 1\frac{3}{7}+\frac{1}{3}\text{ of }2\frac{1}{4}-\frac{13}{28}\times 2}{\frac{13}{28}\text{ of }2+\frac{1}{3}\text{ of }2\frac{1}{4}-1\frac{1}{4}\text{ of }1\frac{3}{7}}$.

21. $2\frac{1}{4}\times\dfrac{10\frac{3}{4}-4\frac{11}{12}}{6\frac{5}{16}+7\frac{2}{3}}\times\dfrac{3\frac{5}{11}}{1\frac{2}{3}\times 9\frac{1}{11}}$.

22. $\dfrac{8\frac{7}{8}-7\frac{6}{7}+5\frac{5}{6}-4\frac{4}{5}}{9\frac{9}{10}-8\frac{13}{15}+7\frac{7}{8}-6\frac{6}{7}}$.

TO EXPRESS ONE NUMBER AS THE FRACTION OF ANOTHER.

275. What fraction of 8 is 7? *Ans.* $\frac{7}{8}$.

For since $1 = \frac{1}{8}$ of 8,
$7 = 7$ *times* $\frac{1}{8}$ of 8;
that is, $7 = \frac{7}{8}$ of 8.

It will be noticed that the number which follows "of" is the denominator, and the other number the numerator, of the required fraction. See, also, § 166.

EXERCISE XXVIII.

What fraction of

1. 8 is 3?
2. 3 is 8?
3. 9 is 7?
4. 7 is 9?
5. 8 is 12?
6. 12 is 8?
7. $2\frac{1}{5}$ is $\frac{3}{5}$?
8. $\frac{3}{5}$ is $2\frac{1}{5}$?
9. $2\frac{3}{4}$ is $1\frac{1}{4}$?
10. $1\frac{1}{4}$ is $2\frac{3}{4}$?
11. $2\frac{1}{3}$ is $7\frac{1}{8}$?
12. $7\frac{1}{3}$ is $2\frac{1}{8}$?
13. $3\frac{1}{3}$ is $8\frac{1}{7}$?
14. \$2 is \$$1\frac{1}{2}$?
15. \$$2\frac{1}{2}$ is \$5?
16. \$$\frac{3}{4}$ is \$$\frac{1}{4}$?
17. \$$\frac{5}{4}$ is \$$\frac{3}{8}$?
18. \$$2\frac{3}{4}$ is \$$\frac{1}{3}$?
19. \$$\frac{1}{2}$ is \$$\frac{1}{10}$?
20. \$1 is \$$\frac{7}{8}$?
21. \$10 is \$$\frac{2}{3}$?
22. \$100 is \$6?
23. \$100 is \$$4\frac{1}{2}$?
24. \$4 is \$25?
25. $100\frac{5}{9}$ is $8\frac{4}{9}$?
26. 21 is $\frac{15}{19}$ of $3\frac{4}{5}$?
27. $18\frac{117}{256}$ is $\frac{5}{8}$ of $33\frac{3}{4}$?
28. $3\frac{1}{3}$ is $\frac{2}{3} \times 1\frac{1}{8}$?
29. $3\frac{1}{11} \times 5\frac{1}{27}$ is 1720?
30. $3\frac{1}{2} \times \frac{8}{9}$ of $\frac{4}{7}$ is $1\frac{3}{8}$?

What part of

31. $\frac{28}{59} \times \frac{59}{68}$ is $\frac{1}{6} \times 4 \times \frac{2}{3}$?
32. $13\frac{5}{8} \times \frac{2}{3} \times \frac{9}{65}$ is $\frac{2}{3}$ of $1\frac{44}{65}$ of $1\frac{1}{8}$?
33. $\frac{17}{20} + \frac{11}{15} + \frac{7}{10} + \frac{4}{5}$ is $\frac{17}{20} - \frac{11}{15} + \frac{7}{10} - \frac{4}{5}$?
34. $4\frac{1}{7} - 2\frac{1}{4}$ is $6\frac{1}{2} - 2\frac{1}{7}$?
35. $17\frac{2}{3} - 12\frac{6}{7}$ is $5 - \frac{1}{18} - \frac{4}{39} - \frac{1}{25}$?
36. $24 - 17\frac{4}{13}$ is $7 + \frac{2}{15} - \frac{5}{81} - \frac{11}{45}$?
37. $\frac{3}{7}$ of $2\frac{1}{17}$ is $1\frac{2}{3} \div 2\frac{3}{7}$?
38. $\left(\frac{7}{4-\frac{5}{6}} - \frac{5}{6-\frac{3}{8}}\right) \div \left(\frac{4}{7-\frac{4}{7}} + \frac{2}{4-\frac{2}{5}}\right)$ is $\left(14 - \frac{1}{\frac{1}{2}-\frac{6}{31}}\right) \div \left(\frac{1}{\frac{1}{2}-\frac{27}{59}} - 13\right)$?

TO REDUCE A DECIMAL TO A COMMON FRACTION.

(1) Reduce .527 to a common fraction.

.527 means $\frac{5}{10} + \frac{2}{100} + \frac{7}{1000} = \frac{500+20+7}{1000} = \frac{527}{1000}$.

(2) $.525 = \frac{525}{1000} = \frac{21}{40}$.

(3) $18.375 = 18\frac{375}{1000} = 18\frac{15}{40} = 18\frac{3}{8}$.

276. Hence, to reduce a decimal to a common fraction,

Write the figures of the decimal for the numerator; and 1 *with as many zeros as there are figures in the decimal for the denominator.*

Exercise XXIX.

Reduce to common fractions in their lowest terms:

1.	.125.	**9.**	7.015625.	**17.**	125.6048.
2.	.625.	**10.**	20.100256.	**18.**	.128.
3.	.675.	**11.**	10.012575.	**19.**	.73125.
4.	10.864.	**12.**	104.335.	**20.**	1.1875.
5.	50.84.	**13.**	50.0004.	**21.**	.603125.
6.	3.00025.	**14.**	100.001.	**22.**	6.03125.
7.	8.1075.	**15.**	8.00725.	**23.**	60.3125.
8.	35.01024.	**16.**	20.018375.	**24.**	7.0315.

To Reduce a Common Fraction to a Decimal.

(1) Change $\frac{5}{8}$ to a decimal, and $\frac{2}{3}$ to a decimal.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \end{array} \qquad \begin{array}{r} 0.66667 \\ 3\overline{)2.00000} \end{array}$$

Carry the quotient to five decimal places unless it terminates before the fifth place.

277. Hence, to reduce a common fraction to a decimal,

Divide the numerator by the denominator.

Exercise XXX.

Reduce to decimals:

1.	$\frac{7}{8}$.	**6.**	$4\frac{11}{800}$.	**11.**	$\frac{17}{4000}$.	**16.**	$\frac{124}{16}$.
2.	$\frac{15}{16}$.	**7.**	$5\frac{5}{32000}$.	**12.**	$\frac{119}{128}$.	**17.**	$\frac{2}{3}$ of $1\frac{4}{5}$.
3.	$\frac{9}{32}$.	**8.**	$9\frac{128}{25600}$.	**13.**	$\frac{13}{625}$.	**18.**	$\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{7}{10}$.
4.	$\frac{9}{25}$.	**9.**	$11\frac{19}{4000}$.	**14.**	$\frac{11}{256}$.	**19.**	$3\frac{3}{5}$ of $4\frac{1}{9}$.
5.	$\frac{5}{64}$.	**10.**	$\frac{9}{125}$.	**15.**	$\frac{3}{160}$.	**20.**	$\frac{29}{32}$ of $\frac{49}{64}$.

CHAPTER XII.

MEASURES IN COMMON USE.

278. The unit of measure for lengths is a **yard**; and from this are derived the units of surface and volume.

279. The **standard yard** of Great Britain, as defined by act of Parliament, is the distance between the centres of two cylindrical holes in a certain bar of gun-metal, when the metal has a temperature of 62 degrees Fahrenheit.

280. The standard yard of the United States conforms as nearly as possible to that of Great Britain.

LINEAR MEASURES.

281. 12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
5½ yards, or 16½ feet = 1 rod (rd.).
320 rods, 1760 yards, or 5280 feet = 1 mile (mi.).

A line = $\frac{1}{12}$ in.
A barleycorn = ⅓ in.
A hand (used in horse measure) = 4 in.
A palm = 3 in.
A span = 9 in.
A cubit = 18 in.
A military pace = 2½ ft.
A chain = 4 rds.
A link = $\frac{1}{100}$ chain.
A furlong = ⅛ m.
A knot (used in navigation) = 6086 ft.
A league = 3 knots.
A fathom (used in measuring depths at sea) = 6 ft.
A cable-length = 120 fathoms.

282. A quantity expressed with reference to a *single unit* is called a **simple quantity**; but a quantity expressed with reference to *different units* is called a **compound quantity.** Thus, 20 miles is a simple quantity; but 20 mi. 70 rds. 5 ft. is a compound quantity.

NOTE. A number expressed with reference to a particular unit is sometimes called a *denominate*, or *concrete*, number; and, in distinction, a number expressed without reference to a unit is called an *abstract* number. But it is to be observed that all *numbers* are abstract.

(1) Change 20 mi. to rods.

$$\begin{array}{r} 320\text{ rds.} \\ \underline{20\phantom{\text{ rds.}}} \\ 6400\text{ rds.} \end{array}$$

Since a mile contains 320 rds., 20 miles contain twenty *times* 320 rds. = 6400 rds.

(2) Change 20 mi. 70 rds. 5 ft. to feet.

$$\begin{array}{r} 20\text{ mi. } 70\text{ rds. } 5\text{ ft.} \\ \underline{320} \\ 6400 \\ \underline{70} \\ 6470\text{ rds.} \\ \underline{16\frac{1}{2}} \\ 3235 \\ 38820 \\ \underline{6470} \\ 106755 \\ \underline{5} \\ 106760\text{ ft.} \end{array}$$

20 times 320 rds. = 6400 rds., to which the 70 rds. are added. Again, 6470 times 16½ ft. = 106,755 ft., to which the 5 ft. are added.

Observe that multiplicand and multiplier are interchanged in the operation.

(3) Change 6400 rds. to miles.

$$\begin{array}{r} 320\overline{)6400} \\ 20\text{ mi.} \end{array}$$

Since there are 320 rds. in 1 mi., in 6400 rds. there are (6400 ÷ 320) mi. = 20 mi.

(4) Change 106,760 ft. to miles.

$$\begin{array}{r l}
16\frac{1}{2}\,)\,106760 \text{ ft.} & \\
2 & \\
33\overline{)\,213520} & \ldots\ldots\ \text{half-feet.}\\
320\overline{)\,6470} \text{ rds.} & \ldots\ 10 \text{ half-feet} = 5 \text{ ft.}\\
20 \text{ mi.} & \ldots\ 70 \text{ rds.}
\end{array}$$

20 mi. 70 rds. 5 ft. *Ans.*

There are 16½ ft., or 33 half-feet, in a rod; so the 106,760 ft. are changed to half-feet, and *these* to rods, by dividing by 33. The remainder is 10 half-feet = 5 ft.

6470 rds. are changed to miles by dividing by 320, the number of rods in a mile. The remainder is 70 rds.

283. The process of changing the *unit* in which a quantity is expressed, without changing the *value* of the quantity, is called **reduction.**

284. If the change be from a higher denomination to a lower, it is called *reduction descending;* if from a lower to a higher, it is called *reduction ascending.*

285. To reduce a higher denomination to a lower,

Multiply by the number required of the lower denomination to make one of the higher.

If there be intermediate denominations, reduce step by step, and add to each result the given number of the denomination reached.

286. To reduce a lower denomination to a higher,

Divide by the number required of the lower denomination to make one of the higher.

If there be intermediate denominations, reduce step by step.

EXERCISE XXXI.

Reduce:

1. 3 yds. 2 ft. to inches.
2. 4 mi. 124 rds. to feet.
3. 27 rds. 4½ yds. to inches.
4. 290 leagues to feet.
5. 82,976,432 in. to miles.
6. 7 mi. 3½ yds. to inches.
7. 27 mi. 222 rds. to inches.
8. 712 mi. to inches.
9. 540,451 ft. to miles.
10. 271,256 in. to miles.
11. 723,964 ft. to miles.
12. 233,205 in. to miles.
13. 10 chains to inches.
14. 233,185 in. to fathoms.
15. If the height of a horse be 16 hands, how many feet is his height?
16. If a train move 40 ft. in a second, what is its rate in miles per hour? (One hour = 3600 seconds.)

287.

MEASURES OF SURFACE.

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
30¼ square yards, or 272¼ square feet = 1 square rod (sq. rd.).
160 square rods, or 10 square chains = 1 acre (A.).
640 acres = 1 square mile (sq. mi.).

A square of flooring or roofing = 100 sq. ft.
A section of land = 1 mile square.
A township = 36 sq. mi.

NOTE. The units of surface measure, except the acre, are *squares*, the sides of which are *linear units*. Thus, a square inch is a square each side of which is one inch in length.

Hence, in surface measurement, the scale ascends and descends by *squares* of the linear units; $144=12^2$; $9=3^2$; $30\frac{1}{4}=(5\frac{1}{2})^2$; $272\frac{1}{4}=(16\frac{1}{2})^2$.

Reduce 740 sq. yds. to sq. rds.

$30\frac{1}{4}$)740 sq. yds.
4
121)2960 quarter-yards.
24 sq. rds. . . . 56 quarter-yards = 14 sq. yds.

$30\frac{1}{4}$ sq. yds. made a square rod; that is, 121 quarter-yards make a square rod. Therefore, the 740 sq. yds. are reduced to quarter-yards; and these quarter-yards to square rods, by dividing by 121. The remainder, 56, is 56 quarter-yards = 14 yds.

EXERCISE XXXII.

Reduce.

1. 92,638 sq. yds. to inches.
2. 1,223,527 sq. in. to yards.
3. 721 sq. mi. to rods.
4. 34,729 sq. yds. to rods.
5. 3 A. 107 sq. rds. 27 sq. yds. 7 sq. ft. 23 sq. in. to inches.
6. 1 A. to feet.
7. 99,894,712 sq. in. to acres.
8. 15,376 sq. yds. to acres.
9. 562,934 sq. in. to rods.

288.

MEASURES OF VOLUME.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).
128 cubic feet = 1 cord (cd.).

40 feet of round timber, or 50 feet of hewn timber = 1 ton, or load.
40 cubic feet = 1 ton of shipping.

NOTE I. A pile of wood 4 ft. wide, 4 ft. high, and 8 ft. long, is a *cord*. One foot of the length of such a pile is called a *cord foot*, and contains 16 cubic feet.

NOTE II. The units of volume, except the cord, are *cubes*, the edges of which are *linear units*. Thus, a cubic inch is a cube each edge of which is one inch in length. Hence, in the measurement of volumes the scale ascends and descends by *cubes* of the linear units; thus $1728 = 12^3$; $27 = 3^3$.

EXERCISE XXXIII.

Reduce:

1. 7 cu. yds. 13 cu. ft. to cubic feet.
2. 25 cu. yds. 5 cu. ft. 143 cu. in. to cubic inches.
3. 74,325 cu. in. to cubic feet.
4. 439,000 cu. in. to cubic yards.
5. 921,730 cu. in. to cubic yards.
6. Wood cut in lengths of 4 ft. is piled to a height of 3½ ft. How long must the pile be to contain a cord?
7. A pile of wood 127 ft. long, 4 ft. wide, and 3 ft. 8 in. high is sold for $7 a cord. How much money is received for it?

MEASURES OF CAPACITY.

Dry Measure.

289.

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

Liquid Measure.

290.

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

31½ gallons = 1 barrel (bbl.).
2 barrels = 1 hogshead (hhd.).

NOTE. The gallon of liquid measure contains 231 cubic inches. The bushel of dry measure contains 2150.42 cubic inches. Therefore the quart of liquid measure contains 57¾ cubic inches, and the quart of dry measure 67⅕ cubic inches. The imperial gallon of *Great Britain* contains 277.274 cubic inches. The imperial bushel contains 2218.192 cubic inches. A quarter contains 8 imperial bushels.

EXERCISE XXXIV.

Reduce:

1. 3 pks. 5 qts. 1 pt. to pints.
2. 4234 pts. to bushels.
3. 24 gals. 2 qts. 1 pt. 2 gi. to gills.
4. 272 liquid quarts to dry quarts.
5. 400 dry quarts to liquid quarts.
6. Express a bushel in cubic feet, carrying the decimal to three places.
7. Express a cubic foot as the decimal fraction of a bushel.
8. 1715½ bushels to pints.
9. 3047 gals. to barrels.

WEIGHTS.

Troy Weight.

291. 24 grains (grs.) = 1 pennyweight (dwt.).
20 pennyweights = 1 ounce (oz.).
12 ounces = 1 pound (lb.).

A carat (of diamond) = 3⅕ grs.

Avoirdupois Weight.

292. 16 drams (drs.) = 1 ounce (oz.).
16 ounces = 1 pound (lb.).
100 pounds = 1 hundred-weight (cwt.).
20 hundred-weight = 1 ton (t.).

112 pounds = 1 long hundred-weight.
2240 pounds = 1 long ton.

NOTE. Troy weight is used for weighing gold, silver, and precious stones. The troy pound is the standard unit of weight.

Avoirdupois weight is used for weighing all other articles.

In the United States custom house and in wholesale transactions in coal and iron the long ton is used.

The pound troy contains 5760 grains.

The pound avoirdupois contains 7000 troy grains.

In preparing medicines, apothecaries use the following:

Apothecaries' Weight.

293. 20 grains (grs.) = 1 scruple (℈).
3 ℈ = 1 dram (ʒ).
8 ʒ = 1 ounce (℥).
12 ℥ = 1 lb. troy.

Apothecaries' Measure.

294. 60 minims (♏) = 1 dram (♏ lx.).
8 drams = 1 ounce (fl. drm. viij.).
16 ounces = 1 pint (fl. oz. xvj.).

EXERCISE XXXV.

Reduce:

1. 27,587 grs. to pounds troy.
2. 34,652 lbs. to long tons.
3. 136,851 oz. to tons.
4. 864,205 grs. (troy) to pounds.
5. 864,205 grs. (apoth.) to pounds avoirdupois.
6. 5 lbs. 7 oz. 6 dwts. 12 grs. to grains.
7. 745 lbs. avoirdupois to troy weight.
8. 745 lbs. troy to avoirdupois weight.
9. 23,047,125 drs. to tons.
10. 90,252,381 drs. to tons.
11. 1 pt. to minims.
12. 8000 ♏ to ounces.

295. TIME.

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (dy.).
7 days = 1 week (wk.).
365 days (or 52 wks. 1 dy.) = 1 common-year (yr.).
366 days = 1 leap-year.
100 years = 1 century.

NOTE. The interval measured from the instant the sun is due south until it is due south the next day, is called a *solar day.* Its length varies slightly, and its average length, called the *mean solar day*, is the day of which an hour is the twenty-fourth part.

A *year* is the time in which the earth performs one revolution around the sun, and consists of 365.242218 mean solar days.

Before the time of Julius Cæsar, the year was reckoned as 365 days. On the supposition that 365¼ days was the true length, he introduced a calendar in which every fourth year (*every year which will give an integral quotient when its number is divided by* 4) was to consist of 366 days. The year of 366 days is called a *leap-year.*

The error of the Julian calendar, (365.25 – 365.242218 =) .007781 of a day, would amount to 3.1124 days in four centuries. To correct this error Pope Gregory XIII., in 1582, introduced a calendar in which three leap-years in every four centuries were reckoned as common years. Hence, the centuries are not leap-years unless *the number of the century* divided by 4 gives an integral quotient.

The year is divided into twelve *calendar months.* Four months, April, June, September, and November, have each thirty days; February has twenty-eight days, and in leap-year twenty-nine; the rest have each thirty-one days.

A *lunar month* is the time between two new moons, and is a little more than 29 dys. 12 hrs. 44 min.

EXERCISE XXXVI.

Reduce:

1. 6 hrs. 17 min. 25 sec. to seconds.
2. 1 yr. 13 dys. 4 min. to minutes.
3. 48,567 min. to days.
4. 742,392 sec. to days.
5. Find the number of days, reckoning from noon of the one to noon of the other, between the following days in the year 1880: July 4 and December 2; February 1 and May 29; January 3 and October 15; also, between December 25, 1880, and May 25, 1881.
6. How many minutes are there from midnight of March 7 to midnight of June 20?
7. Find the number of seconds from eight o'clock Monday morning till six o'clock the next Saturday evening.
8. Which of the years 1600, 1656, 1700, 1734, 1800, 1818, 1880, 1900, 1924, 2000 are leap years?

CIRCULAR AND ANGULAR MEASURES.

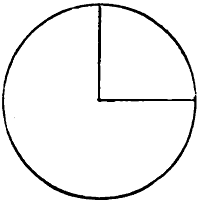
296. The circumference of every circle is divided into 360 equal parts, called degrees; each degree into 60 minutes, and each minute into 60 seconds.

297. 60 seconds (″) = 1 minute (′).
60′ = 1 degree (°).
360° = 1 circumference.

NOTE. As the circumference of every circle has 360 degrees, the length of the degree differs in different circles. A degree of the circumference of the earth at the equator contains 60 geographical miles, or 69.16 statute miles.

If two lines drawn from the centre of a circle include *one-fourth* of the circumference, they form at the centre an angle of 90°, and this is called a right angle. The lines are said to be perpendicular to each other.

The number of degrees in *the angle* formed by two lines drawn from the centre of a circle is the same as the number of degrees which the lines intercept on *the circumference.*



EXERCISE XXXVII.

Reduce:

1. 2° 30′ 25″ to seconds.
2. 15° 3′ 22″ to seconds.
3. 56,760″ to degrees.
4. 212,221″ to degrees.
5. The hour and minute hands of a watch form an angle of how many degrees at 3 o'clock? at 4 o'clock? at 6 o'clock? at 7½ o'clock? at 11 o'clock? at 12 o'clock?
6. How many geographical miles in the width of the torrid zone (46° 55′)? How many statute miles?

MONEY.

For United States Money see § 182.

English Money.

298. 4 farthings = 1 penny (*d.*).
12 *d.* = 1 shilling (*s.*).
20 *s.* = 1 pound (£).

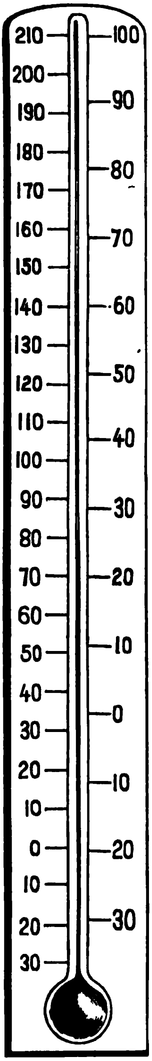
A guinea = 21 shillings.
A crown = 5 *s.*
A half-crown = 2 *s.* 6 *d.*
A florin = 2 *s.*
A sovereign = 20 *s.*
A half-sovereign = 10 *s.*
A sovereign = $4.866½.

French Money: 100 centimes = 1 franc (fr.) = $0.193.
German Money: 100 pfennigs = 1 mark = $0.238.
Russian Money: 100 kopecks = 1 rouble = $0.734.
Austrian Money: 100 kreutzers = 1 florin (fl.) = $0.453.

EXERCISE XXXVIII.

Reduce:

1. £583 6 *s.* 8 *d.* to pence.
2. £79 18 *s.* 11½ *d.* to farthings.
3. 28,572 *d.* to pounds.
4. 272,191 *d.* to half-sovereigns.
5. 27,281 crowns to guineas.
6. 1,716,114 guineas to pounds.
7. 291,374 far. to pounds.
8. 709,126 *d.* to guineas.
9. 286,347 far. to crowns.
10. 20 francs to dollars.
11. 20 marks to dollars.
12. 5 roubles to dollars.
13. Find the whole sum of money in a box containing 35 sovereigns, 27 half-sovereigns, 13 crowns, 41 half-crowns, and 85 shillings.



TEMPERATURE.

299. There are three scales for registering temperature by means of the thermometer.

300. Fahrenheit's has the freezing point of water marked 32°, and boiling point 212°.

301. The Centigrade has the freezing point zero, and the boiling point 100.

302. Reaumur's has the freezing point zero, and the boiling point 80.

303. The reduction from one scale to another may be effected as follows:

(1) Express 80° C. in Reaumur's scale.

100° C. = 80° R.
Therefore, 80° C. = $\frac{80}{100}$ of 80° R.
= 64° R.

(2) Express 50° F. in Centigrade scale.

The number of degrees F. between the freezing and boiling points is 180;
Therefore, 1° F. = $\frac{100}{180}$ C. = $\frac{5}{9}$° C.
But 50° F. is 180° F. above freezing point,
and $\frac{5}{9}$ of 18° = 10°.
That is, 50° F. = 10° C.

(3) Express 60° C. on Fahrenheit's scale.

100° C. = 180° F.
Therefore, 60° C. = $\frac{60}{100}$ of 180° F.
= 108° F.

This will be the height above the freezing point, and will be marked 32° + 108° = 140°.

EXERCISE XXXIX.

Express:

1. 59° F. in Centigrade scale; in Réaumur's scale.
2. 77° F. in Centigrade scale; in Réaumur's scale.
3. 950° F. in Centigrade scale; in Réaumur's scale.
4. −40° F. in Centigrade scale; in Réaumur's scale.
5. −4° F. in Centigrade scale; in Réaumur's scale.
6. 10° C. in Fahrenheit's scale; in Réaumur's scale.
7. 22° C. in Fahrenheit's scale; in Réaumur's scale.
8. −30° C. in Fahrenheit's scale; in Réaumur's scale.
9. $-11\frac{3}{7}$° C. in Fahrenheit's scale; in Réaumur's scale.
10. 4° R. in Fahrenheit's scale; in Centigrade scale.
11. 12° R. in Fahrenheit's scale; in Centigrade scale.
12. −20° R. in Fahrenheit's scale; in Centigrade scale.
13. 4° C. in Fahrenheit's scale; in Réaumur's scale.
14. 0° F. in Centigrade scale; in Réaumur's scale.

MISCELLANEOUS.

Numbers.

304. 12 units = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 units = 1 score.

Paper.

24 sheets = 1 quire.
20 quires = 1 ream.
2 reams = 1 bundle.
5 bundles = 1 bale.

Weights.

A bushel of corn or rye = 56 lbs.
A bushel of corn meal, rye meal, or cracked corn = 50 lbs.
A bushel of wheat = 60 lbs.
A bushel of potatoes = 60 lbs.
A bushel of beans = 60 lbs.
A bushel of oats = 32 lbs.

A bushel of barley = 48 lbs.
A bushel of timothy-seed = 45 lbs.
A stone of iron or lead = 14 lbs.
A pig of iron or lead = $21\frac{1}{2}$ stone.
A fother of iron or lead = 8 pigs.

The weight of a bushel of potatoes, corn, etc., varies slightly in different States, but the weights here given are those generally adopted in business transactions.

A barrel of flour = 196 lbs.
A barrel of pork or beef = 200 lbs.
A cask of lime = 240 lbs.
A cental of grain = 100 lbs.
A quintal of fish = 100 lbs.

Books.

A book formed of sheets folded in

2 leaves is a folio;
4 leaves is a quarto;
8 leaves is an octavo;
12 leaves is a duodecimo;
16 leaves is a 16mo.

COMPOUND ADDITION.

305. Compound Addition is the operation of finding the sum of two or more compound quantities expressed in the same measure.

	wks.	dys.	hrs.	min.
Ex.	13	3	6	20
	8	6	14	34
	11	4	20	28
	5	2	10	56
	39	3	4	18

Write the numbers so that units of the same denomination shall be in the same column.

The sum of the minutes is 138. Divide 138 min. by 60 (60 min. = 1 hr.).

The result is 2 hrs. 18 min.

Write 18 under the column of minutes, and carry 2 to the column of hours.

The sum of the hours, including the two hrs. obtained from the 138 min., is 52. Divide by 24 (24 hrs. = 1 dy.).

The result is 2 dys. 4 hrs.

Write 4 under the column of hours, and carry 2 to the column of days.

The sum of the days, including the 2 dys. obtained from the 52 hrs., is 17. Divide by 7 (7 dys. = 1 wk.).

The result is 2 wks. 3 dys.

Write 3 under the column of days, and carry 2 to the column of weeks.

The sum of the weeks, including the 2 wks. from the 17 dys., is 39.

	mi.	rds.	yds.	ft.		
(2)	4	110	5	2		
	6	25	4	2		
	10	136	4½	1		
			½=	1	6 in.	
	10	136	4	2	6 in.	*Ans.*

In the result, the ½ yd. is changed to 1 ft. 6 in.

EXERCISE XL.

Add:

	hrs.	min.	sec.
1.	14	21	37
	17	13	32
	9	47	43
	12	53	54
	22	17	50

	cu. yds.	cu. ft.	cu. in.
2.	130	5	820
	56	20	304
	37	4	86
	8	10	129
	12	19	175

	£	s.	d.
3.	35	2	6¾
	18	5	4
	27	3	10
	12	0	5

	mi.	rds.	yds.	ft.	in.
4.	6	120	3	2	2
	18	15	1	1	6
	3	215	2	2	3
	7	95	1	1	8

	A.	sq. rds.	sq. yds.	sq. ft.	sq. in.
5.	74	21	5	4	100
	23	37	13	5	83
	12	106	17	8	7
	41	50	23	0	24

6. 5 bu. 3 pks. 6 qts. 1 pt.; 6 bu. 2 pks. 7 qts.; 7 bu. 1 pk. 1 qt. 1 pt.; 1 pk. 7 qts.; 2 bu. 3 pks. 1 pt.

7. 48 t. 13 cwt. 75 lbs. 6 oz. 4 drms.; 25 t. 12 cwt. 27 lbs. 8 oz. 13 drms.; 51 t. 10 cwt. 44 lbs. 15 drms.; 80 t. 5 cwt. 6 oz.; 19 cwt. 27 lbs.; 25 lbs. 8 oz. 10 drms.; 5 t. 5 cwt. 5 oz.

8. 50 gals. 3 qts. 1 pt. 3 gi.; 12 gals. 1 qt. 1 pt. 1 gi.; 5 gals. 2 qts. 1 pt. 2 gi.; 75 gals. 3 qts. 1 pt. 3 gi.; 80 gals. 3 qts. 0 pts. 1 gi.; 17 gals. 1 qt. 1 pt. 3 gi.

9. 13 lbs. 4 oz. 8 dwt. 6 grs.; 25 lbs. 8 oz. 13 dwt. 20 grs.; 8 lbs. 11 oz. 14 grs.; 20 lbs. 16 dwt. 8 grs.; 15 lbs. 9 oz. 12 dwt.; 4 oz. 3 dwt.

10. 4 gals. 3 qts. 1 pt.; 3 gals. 2 qts. 1½ pts.; 12 gals. 3 qts.; 14 gals. 1½ pts.; 5 gals. 2 qts. 1 pt.
11. 60° 50′ 50″; 20° 41′ 52″; 30° 25′ 20″; 20° 32′ 43″.

COMPOUND SUBTRACTION.

306. Compound Subtraction is the operation of finding the difference between two compound quantities expressed in the same weight or measure.

	£	*s.*	*d.*
Ex.	36	6	8¾
	24	8	3¼
	11	18	5½

Here 6 *s.* is less than 8 *s.*, so £1 is reduced to shillings and added to the 6 shillings. Then 8 *s.* are taken from the 20 *s.* + 6 *s.*, and 1 is added to the £ 24 of the subtrahend.

Exercise XLI.

Subtract:

1. 23 lbs. 8 oz. 19 dwt. 10 grs. from 58 lbs. 6 oz. 17 dwt. 21 grs.
2. 5 bu. 1 pk. 6 qts. 1 pt. from 5 bu. 3 pks. 3 qts.
3. 32 cu. yds. 13 cu. ft. 1600 cu. in. from 39 cu. yds. 17 cu. ft. 1400 cu. in.
4. £92 15 *s.* 1¼ *d.* from £120 13 *s.* 4 *d.*
5. 22 gals. 3 qts. 1 pt. from 30 gals. 2 qts.
6. 17 t. 7 cwt. 17 lbs. 6 oz. 10 drs. from 25 t. 13 cwt. 15 lbs. 12 oz. 5 drs.
7. 13 A. 150 sq. rds. 98 sq. ft. 10 sq. in. from 20 A.
8. 58° 33′ 36″ from 90° 11′ 21″.
9. 2 yrs. 213 dys. 17 hrs. from 3 yrs. 147 dys. 14 hrs.
10. 3 mi. 217 rds. 4 yds. 1 ft. 3 in. from 4 mi. 100 rds. 3 yds.

COMPOUND MULTIPLICATION.

307. Compound Multiplication is the operation of finding the amount of a quantity taken any number of times, when the quantity is expressed in several denominations.

(1) Multiply £13 14 *s.* 7 *d.* by 9.

$$\begin{array}{rrr} \text{£} & s. & d. \\ 13 & 14 & 7 \\ & & 9 \\ \hline 123 & 11 & 3 \end{array}$$

9 × 7 *d.* = 63 *d.* = 5 *s.* 3 *d.*
9 × 14 *s.* = 126 *s.*; and with the 5 *s.* added = 131 *s.* = £6 11 *s.*
9 × £13 = £117; and with the £6 added = £123.

NOTE. If the multiplier be the product of two factors, multiply the given quantity by one of the factors, and the resulting product by the other. Thus, if the multiplier be 72, multiply the given quantity by 8 and the resulting product by 9.

If the multiplier be a large number, the work may be arranged as in the following example:

(2) Multiply 25 lbs. 10 oz. 11 dwt. 16 grs. by 439.

$$\text{I.}\quad \begin{array}{rl} & 439 \\ & \underline{16} \\ & 2634 \\ & 439 \\ 24\left\{\begin{array}{r}4\\6\end{array}\right. & \begin{array}{|r}\hline 7024 \\ \hline 1756 \\ \hline \end{array} \\ & 292 \ldots \tfrac{4}{6}\text{ dwt.} = 16\text{ grs.} \end{array} \qquad \text{II.}\quad \begin{array}{rl} & 439 \\ & \underline{11} \\ & 4829 \\ & \underline{292} \ldots \text{from I.} \\ 20) & 5121 \\ & \overline{256} \ldots 1\text{ dwt.} \end{array}$$

$$\text{III.}\quad \begin{array}{rl} & 439 \\ & \underline{10} \\ & 4390 \\ & \underline{256} \ldots \text{from II.} \\ 12) & 4646 \\ & \overline{387} \ldots 2\text{ oz.} \end{array} \qquad \text{IV.}\quad \begin{array}{rl} & 439 \\ & \underline{25} \\ & 2195 \\ & \underline{878} \\ & 10975 \\ & \underline{387} \ldots \text{from III.} \\ & 11362\text{ lbs.} \end{array}$$

11,362 lbs. 2 oz. 1 dwt. 16 grs. *Ans.*

EXERCISE XLII.

Multiply:

1. £31 2*s.* 6½ *d.* by 8.
2. 19 gals. 3 qts. 1 pt. by 70.
3. 3 lbs. 4 oz. 8 dwt. 10 grs. by 10.
4. 5 t. 10 cwt. 67 lbs. by 10.
5. 43 bu. 2 pks. by 63.

6. 15 wks. 3 dys. 5 hrs. 12 min. by 7.
7. 5 cu. yds. 10 cu. ft. 371 cu. in. by 6.
8. 27 A. 76 sq. rds. 22 sq. yds. 5 sq. ft. by 90.
9. 32 rds. 3 yds. 1 ft. by 57.
10. 34 dys. 10 hrs. 13 min. 12 sec. by 108.
11. 5 mi. 126 rds. 19 yds. 6 in. by 7125.
12. 11 ℥ 5 ʒ 2 ℈ 11 grs. by 2197.

COMPOUND DIVISION.

308. Compound Division is

I. The operation of finding a quantity expressed in several denominations, which, when multiplied by a given number, shall equal a given quantity of the same kind.

II. The operation of finding the number of times that a quantity expressed in several denominations is contained in another quantity of the same kind.

NOTE. It is to be observed that the result of the division in I. is a quantity of the same kind as the dividend; in II. it is an abstract number.

(1) Divide £28 11*s.* 8*d.* by 7.

$$\begin{array}{r|rrr} & \pounds & s. & d. \\ 7 & 28 & 11 & 8 \\ \hline & 4 & 1 & 8 \end{array}$$

The remainder from dividing the shillings by 7 is 4 *s.* = 48 *d.*, which, with the 8 *d.* added, is 56 *d.*; and this divided by 7 gives 8 *d.*

(2) Divide 40 t. 12 cwt. 20 lbs. 14 oz. by 36.

$$36\left\{\begin{array}{r|rrrr} & t. & cwt. & lbs. & oz. \\ 4 & 40 & 12 & 20 & 14 \\ \hline 9 & 10 & 3 & 5 & 3\frac{1}{2} \\ \hline & 1 & 2 & 56 & 2\frac{1}{6} \end{array}\right.$$

1 t. 2 cwt. 56 lbs. $2\frac{1}{6}$ oz. *Ans.*

(3) Divide 2334 cu. yds. 2 cu. ft. 1693 cu. in. by 649.

cu. yds. cu. ft. cu. in.
649)2334 2 1693(3
1947
387
27
2711
774
649)10451(16
649
3961
3894
67
1728
536
134
469
67
1693
649)117469(181
649
5256
5192
649
649 3 cu. yds. 16 cu. ft. 181 cu. in. *Ans.*

In (3) the remainder from dividing 2334 cu. yds. by 649 is 387 cu. yds., which are reduced to cubic feet by multiplying by 27 (27 cu. ft. = 1 cu. yd.). The result, with the 2 cu. ft. added, is 10451 cu. ft. The remainder from dividing 10451 cu. ft. by 649 is 67 cu. ft., which are reduced to cubic inches by multiplying by 1728 (1728 cu. in. = 1 cu. ft.). The result, with the 1693 cu. in. added, is 117469 cu. in.

(4) Divide £26 11*s.* by 3*s.* 8*d.*

Reduce both quantities to pence.

£26 11*s.* = 6372*d.*
3*s.* 8*d.* = 44*d.*

44)6372
$144\frac{9}{11}$
$144\frac{9}{11}$. *Ans.*

EXERCISE XLIII.

Divide:

1. 54 mi. 124 rds. 1 yd. 2 ft. 6 in. by 33.
2. 5 cu. yds. 1 cu. ft. 84 cu. in. by 1716 cu. in.
3. 8426 wks. 6 dys. 21 hrs. 10 min. 21 sec. by 1029.
4. £394 2*s.* $10\frac{1}{2}$ *d.* by £5 2*s.* $4\frac{1}{2}$ *d.*
5. 22 wks. 2 dys. by 11 hrs. 31 min. 12 sec.
6. 74,128 sq. mi. 517 A. 80 sq. rds. by 10,000.
7. 38° 37′ 42″ by 5° 31′ 6″.

TO EXPRESS THE VALUE OF THE FRACTION OF A SIMPLE QUANTITY IN LOWER DENOMINATIONS.

309. (1) Express $\frac{2}{3}$ rd. in yards, feet, and inches.

$\frac{2}{3}$ rd. $= \frac{2}{3}$ of $5\frac{1}{2}$ yds. $= 3\frac{2}{3}$ yds.
$\frac{2}{3}$ yds. $= \frac{2}{3}$ of 3 ft. $= 2$ ft.

3 yds. 2 ft. *Ans.*

(2) Find the value of .3975 of a mile.

$$\begin{array}{rl} .3975 & \text{mi.} \\ 320 & \\ \hline 79500 & \\ 11925 & \\ \hline 127.2 & \text{rds.} \\ 16\frac{1}{2} & \\ \hline 3.3 & \text{ft.} \\ 12 & \\ \hline 3.6 & \text{in.} \end{array}$$

In (2) the multiplicand and multiplier are interchanged.
.3975 of 320 rds. = 127.2 rds.
.2 of a rod = 3.3 ft., and
.3 of a foot = 3.6 in.

127 rds. 3 ft. 3.6 in. *Ans.*

EXERCISE XLIV.

Find the value of:

1. $\frac{4}{5}$ of a mile.
2. $\frac{3}{16}$ of an acre.
3. $\frac{5}{8}$ of a hundred-weight.
4. $\frac{2}{3}$ of a pound sterling.
5. $\frac{2}{11}$ of a mile.
6. $\frac{7}{11}$ of an acre.
7. $\frac{4}{9}$ of a degree.
8. $\frac{1}{3}$ of a year.
9. 0.15625 of a bushel.
10. 0.625 of a gallon.
11. 0.875 of a leap-year.
12. 0.325 of a pound troy.

TO FIND THE VALUE OF THE FRACTION OF A COMPOUND QUANTITY.

310. (1) Find the value of $\frac{3}{8}$ of £3 2*s.* 4*d.*

$$\begin{array}{r|rrr} & £ & s. & d. \\ & 3 & 2 & 4 \\ & & & 3 \\ \hline 8 & 9 & 7 & 0 \\ \hline & 1 & 3 & 4\frac{1}{2} \end{array}$$

£1 3*s.* 4½*d.* *Ans.*

(3) Find the value of $2\frac{5}{6}$ of £13 4*s.* 9*d.*

	£	s.	d.		£	s.	d.
	13	4	9		13	4	9
			5				2
6)	66	3	9		26	9	6
	11	0	$7\frac{1}{2}$		11	0	$7\frac{1}{2}$
					37	10	$1\frac{1}{2}$

£37 10*s.* $1\frac{1}{2}$*d.* *Ans.*

NOTE. When the multiplier is a mixed number, multiply by the integer and the fractional parts separately, and add the results.

EXERCISE XLV.

Find the value of:

1. $6\frac{2}{5}$ of 3 A. $101\frac{1}{3}$ sq. rds.
2. $1\frac{3}{7}$ of 7 hrs. 21 min. 27 sec.
3. 10.0175 of 1 dy. 13 hrs.
4. $17\frac{7}{12}$ of 10 yds. 2 ft. $3\frac{1}{5}$ in.
5. 0.01284 of 14 mi.
6. 0.42776 of 12 t. 10 cwt.
7. $\frac{2}{5}$ of 1 lb. + $3\frac{2}{9}$ oz. + $5\frac{2}{3}$ dwt.
8. 0.35 of 4 lbs. 5 oz. 6 dwt. 16 grs.
9. 3.726 mi. – 33.57 rds.
10. $\frac{3}{73}$ of a year + $\frac{9}{56}$ of a week + $\frac{7}{12}$ of an hour.
11. 5.268 of 2 dys. + 2.829 of 16 hrs. + 0.9528 of 25 min.
12. $\frac{3}{16}$ of a mile + $\frac{2}{3}$ of 40 rds. + $\frac{3}{8}$ of a yd.
13. $\frac{3}{4}$ of 2 cwt. 84 lbs. + $\frac{3}{7}$ of 5 cwt. 98 lbs. + $\frac{2}{3}$ of $7\frac{1}{2}$ lbs.

TO EXPRESS ONE QUANTITY AS THE FRACTION OF ANOTHER.

311. (1) Express 10 hrs. 33 min. 36 sec. as the fraction of a day.

$$36 \text{ sec.} = \tfrac{36}{60} \text{ min.} = \tfrac{3}{5} \text{ min.}$$

$$33\tfrac{3}{5} \text{ min.} = \frac{33\frac{3}{5}}{60} \text{ hr.} = \tfrac{168}{300} \text{ hr.} = \tfrac{14}{25} \text{ hr.}$$

$$10\tfrac{14}{25} \text{ hrs.} = \frac{10\frac{14}{25}}{24} \text{ dy.} = \tfrac{264}{600} \text{ dy.} = \tfrac{11}{25} \text{ dy.}$$

$\frac{11}{25}$ dy. *Ans.*

(2) Express 5 yds. 1 ft. 4 in. as the fraction of 1 yd. 2 ft. 4 in.

5 yds. 1 ft. 4 in.:	1 yd. 2 ft. 4 in.:
4 in. $= \frac{4}{12}$ ft. $= \frac{1}{3}$ ft.	4 in. $= \frac{4}{12}$ ft. $= \frac{1}{3}$ ft.
$1\frac{1}{3}$ ft. $= \frac{1\frac{1}{3}}{3}$ yd. $= \frac{4}{9}$ yd.	$2\frac{1}{3}$ ft. $= \frac{2\frac{1}{3}}{3}$ yd. $= \frac{7}{9}$ yd.
$5\frac{4}{9}$ yds.	$1\frac{7}{9}$ yds.

$$\frac{5\frac{4}{9}}{1\frac{7}{9}} = \frac{49}{16}. \quad \textit{Ans.}$$

Exercise XLVI.

Express:

1. A pound avoirdupois as the fraction of a pound troy.
2. An ounce avoirdupois as the fraction of an ounce troy.
3. 363 sq. yds. as the fraction of an acre.
4. $\frac{3}{5}$ of £2 1 *s.* 3 *d.* + $\frac{5}{11}$ of £1 4 *s.* 9 *d.* as the fraction of £2 14 *s.*
5. 2 mi. 138 rds. 1 yd. as the fraction of 3 mi. 265 rds. 3 yds. 1 ft. 6 in.
6. $\frac{2}{7}$ of 560 lbs. as the fraction of 5 long tons.
7. $\frac{2}{3}$ of 200 rds. as the fraction of 4 miles.
8. $\frac{10}{27}$ of 2 dys. 2 hrs. 24 min. as the fraction of 2 wks. 1 d.
9. $\frac{4}{5}$ of the difference between 3 yds. 2 ft. 11 in. and 10 yds. 7 in. as the fraction of 8 yds.
10. $\frac{10}{21}$ of the difference between $\frac{5}{8}$ of 7 hrs. and $\frac{7}{25}$ of 15 min. as the fraction of 12 hrs. 18 min.
11. $\frac{2}{5}$ pt. as the fraction of a gallon.
12. What part of 4 lbs. 1 oz. 8 dwt. 15 grs. is 1 lb. 1 oz. 9 dwt. 15 grs.?
13. What part of 2 mi. is $\frac{2}{3}$ of 6 rds. 3 yds. 2 in.?
14. What part of a bushel is 1 pk. 2 qts. 1 pt.?
15. What part of 20 acres are 19 A. 3.5 sq. ch.?
16. What part of 5 tons are 3 t. 240 lbs.?
17. 38 sq. rds. 194 sq. ft. 108 sq. in. = what part of an acre?

TO EXPRESS ONE QUANTITY AS THE DECIMAL OF ANOTHER.

312. (1) Express 1 oz. 8 dwt. 19.2 grs. as the decimal of a pound.

$$\begin{array}{r|l} 24 & 19.2\ \text{grs.} \\ 20 & 8.8\ \text{dwt.} \\ 12 & \underline{1.44}\ \text{oz.} \\ \multicolumn{2}{l}{\textit{Ans.}\ .12\ \text{lb.}} \end{array}$$

19.2 grs. = .8 dwt., which is written at the right of the 8 dwt.

8.8 dwt. = .44 oz., which is written at the right of the 1 oz.

1.44 oz. = .12 lb.

(2) Express 132 sq. yds. 2 sq. ft. 36 sq. in. as the decimal of 42 sq. yds. 4 sq. ft. 72 sq. in.

$$\begin{array}{r|l} 144 & 36.00\ \text{sq. in.} \\ 9 & \underline{2.25}\ \text{sq. ft.} \\ & 132.25\ \text{sq. yds.} \end{array} \qquad \begin{array}{r|l} 144 & 72.00\ \text{sq. in.} \\ 9 & \underline{4.5}\ \ \text{sq. ft.} \\ & 42.5\ \ \text{sq. yds.} \end{array}$$

$$\frac{132.25}{42.5} = 3.11176.\ \textit{Ans.}$$

36 sq. in. = .25 sq. ft., which is written at the right of the 2 sq. ft.

2.25 sq. ft. = .25 sq. yd., which is written at the right of the 132 sq. yds.

In like manner 42 sq. yds. 4 sq. ft. 72 sq. in. are reduced to square yards and decimal of a yard; then 132.25 sq. yds. are divided by the resulting 42.5 sq. yds.

EXERCISE XLVII.

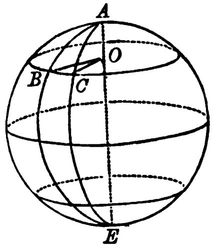
Express:

1. 16*s.* 3¾*d.* as the decimal of a pound.
2. 233 rds. 9 ft. 10.8 in. as the decimal of a mile.
3. 71 sq. rds. 54 sq. ft. 64.8 sq. in. as the decimal of an acre.
4. 15 hrs. 14 min. 6 sec. as the decimal of 2 days.
5. 38 sq. rds. 21 sq. yds. 5 sq. ft. 108 sq. in. as the decimal of an acre.
6. 3 mi. 242 rds. 2 yds. 2 ft. 3 in. as the decimal of 7 mi. 160 rds.
7. 5 hrs. 13 min. 30 sec. as the decimal of a week.
8. 27° 14′ 45″ as the decimal of 90°.

9. 54 dys. 2 hrs. 40 min. as the decimal of $365\frac{1}{4}$ dys.
10. 2 lbs. avoirdupois as the decimal of 10 lbs. troy.
11. 44,920.9025 hrs. as the decimal of a year.
12. 1 drm. avoirdupois as the decimal of 1 dwt. troy.
13. 10 milligrams as the decimal of a grain, if a kilogram equals 2 lbs. 8 oz. 3 dwt. 1 gr.
14. 14.52 sq. yds. as the decimal of a square chain.
15. 8 cwt. 77 lbs. 9.6 oz. as the decimal of a ton.

Longitude and Time.

313. A **meridian** is any line drawn straight around the earth and passing through both poles.



314. The **longitude** of a place is the angle of inclination of the two planes which are supposed to pass through the centre of the earth, and contain, the one the meridian of that place, and the other the standard meridian. Thus, the longitude of *C*, reckoned from meridian *ABE*, is the angle *BOC*. A plane passing through *ABE* divides the earth into Eastern and Western Hemispheres. Places on the Eastern Hemisphere are in East Longitude; on the Western Hemisphere, in West Longitude.

315. As the earth turns upon its axis once in twenty-four hours, a point on the earth's surface will describe a circumference (360°) in twenty-four hours. Therefore longitude may be reckoned in *time* as well as in degrees.

In one hour a point on the earth's surface describes $\frac{1}{24}$ of $360° = 15°$; in one minute $\frac{1}{60}$ of $15° = \frac{1}{4}°$ (or $15'$); and in one second $\frac{1}{60}$ of $15' = \frac{1}{4}'$.

Again, since it requires one hour (60 min.) for a point to pass over 15°, to pass over 1° it requires $\frac{1}{15}$ of 60 min. = 4 min.; and to pass over $1'$ it requires $\frac{1}{60}$ of 4 min. = 4 sec.

(1) Express 17° 35′ 15″ of longitude in time.

$17^\circ\ 35'\ 15''$
$= 17^\circ\ 35.25'$
$= 4 \times (17 \text{ min. } 35.25 \text{ sec.})$
$= 1 \text{ hr. } 10 \text{ min. } 21 \text{ sec.}$
Ans.

Express the longitude in degrees and minutes.

Since it takes 4 × 1 min. to pass over 1°, and 4 × 1 sec. to pass over 1′, it takes 4 × (17 min. 35.25 sec.) to pass over 17° 35.25′.

That is, 1 hr. 10 min. 21 sec.

(2) Express 2 hrs. 13 min. 3.5 sec. in degrees, etc.

$2 \text{ hrs. } 13 \text{ min. } 3.5 \text{ sec.}$
$= 133 \text{ min. } 3.5 \text{ sec.}$
$= \frac{1}{4}\,(133^\circ\ 3.5')$
$= 33^\circ\ 15'\ 52.5''.$
Ans.

Express the time in minutes and seconds.

Since a point passes over ¼ of 1° in 1 min., and ¼ of 1′ in 1 sec., in 133 min. 3.5 sec. it passes over ¼ of (133° 3.5′).

That is, 33° 15′ 52.5″.

Hence, if longitude be expressed in **degrees, multiply** the *number* of degrees by 4; the product is the equivalent *number of minutes of time.*

If longitude be expressed in **minutes of time, divide** the *number* of minutes by 4; the quotient is the equivalent *number of degrees.*

EXERCISE XLVIII.

Find the difference in longitude between two places, if the difference in time be:

1. 1 hr. 15 min.
2. 2 hrs. 11 min.
3. 5 hrs. 10 min. 10 sec.
4. 3 hrs. 25 min. 35 sec.
5. 6 hrs. 12 min. 30 sec.
6. 4 hrs. 8 min. 12 sec.
7. 18 hrs. 10 min.
8. 15 hrs. 15 min. 15 sec.

Find the difference in time between two places, if the difference in longitude be:

9. 9° 20′.
10. 70° 30′.
11. 56° 36′ 12″.
12. 108° 32′ 36″.
13. 120° 14′ 30″.
14. 100° 45′ 54″.
15. 2° 2′ 2″.
16. 75° 10′.

316. Since the sun *appears* to move from **east to west**, sunrise will occur earlier at all points east, and later at all points west, of a given place. Hence, clock-time will be later in all places east, and earlier in all places west, of a given meridian.

Therefore, if the time of a place be given,

To find the time of a place **east, add** to the given time the difference of time between the two places.

To find the time of a place **west, subtract** from the given time the difference of time between the two places.

317. Sun-time, as measured by a sun-dial, counts the hours from the moment that the sun is exactly south at noon. Its hours are slightly unequal in length, being a trifle longer at some seasons than at others.

Clock-time, if measured by a perfect clock, would have all its hours of exactly the same length. Clock-time agrees exactly with sun-time only four times a year; namely, about April 15, June 15, September 1, and December 25.

Local Time is the time of the meridian of the place of which we are speaking.

Standard Time is the clock-time of some selected meridian. Thus, all the railroads, and many of the towns and cities, east of the Alleghanies use Eastern Standard Time; which is the clock-time of a meridian five hours west of the Meridian Telescope in Greenwich, England; this meridian passes a little east of Philadelphia. The railroads between the Alleghanies and the Black Hills use Central Standard Time, exactly one hour slow of the Eastern Standard; it is the clock-time of a meridian passing through New Orleans, and very near Peoria. Mountain Standard Time is the clock-time of the meridian of 105°, and Western Standard Time is the clock-time of 120°.

CHAPTER XIII.

PROBLEMS.

EXERCISE XLIX.

(1) A train travels 24 miles in .8 of an hour. Find its rate per hour.

If the question had been, a train travels 70 mi. in 2 hrs., its rate per hour would obviously be found by dividing the whole distance, 70 miles, by 2. The application of the same method to this question gives,

$$24 \text{ mi.} \div .8 = 30 \text{ mi.} \quad \textit{Ans.}$$

(2) A train runs from New York to Philadelphia, 90 miles, in 1 hr. 33 min. What is its rate per hour?

$$1 \text{ hr. } 33 \text{ min.} = 1\tfrac{11}{20} \text{ hrs.,}$$
$$\text{and } 90 \text{ mi.} \div 1\tfrac{11}{20} = 58\tfrac{2}{31} \text{ mi.} \quad \textit{Ans.}$$

1. A train from New York to Philadelphia, 90 miles, makes the whole distance in 2 hrs. 5 min. What is its rate?
2. Winlock, in 1869, found that electricity went through 7200 miles of wire in $\frac{3}{8}$ of a second. What was its rate per second?
3. If the time required for a signal to pass through the cable from Brest to Duxbury, 3799 miles, be .816 of a second, what is the rate per second?
4. If the report of a gun $1\frac{1}{4}$ miles distant is heard in $5\frac{5}{8}$ seconds after the flash is seen, what is the velocity of sound, in feet, per second?
5. If a man walk $3\frac{1}{5}$ miles in 46 minutes, what is his rate per hour?

6. If a horse go 47½ miles in 10 hrs. 40 min., what is his average rate per hour?
7. If a stone on a glacier move 95½ feet in 188 days, what is its rate, in inches, per day?

It is often required to express speed with reference to the unit of distance instead of the unit of time.

Ex. If a horse made 5½ miles in 33 minutes, how long did it take him to make a mile?

33 min. ÷ 5½ = 6 min. *Ans.*

Hence, when the time is required for the unit of distance, *Divide the time by the distance.*

8. If a horse trot ⅚ of a mile in 2⅛ minutes, in what time can he trot a mile?
9. If a train run 18 miles in 39 minutes, how long does it take to run one mile?
10. If sound travel 1125 feet a second, how long will it take to travel one mile?
11. If a train require 3 hours to travel 104¼ miles, find its average time for travelling a mile.
12. If a mower cut 7½ acres of grass in 3½ days, what part of a day will it take him to cut one acre? If a day consist of 10 working-hours, what part of an acre does he cut in an hour?
13. If a mower cut 3½ square rods in ⅛ of an hour, how many acres can he cut in a day of 10 hours?
14. If a fountain yield 117½ gallons in ¾ of an hour, at what rate per hour is it flowing?
15. If a merchant's profits be $3147 in 7½ months, what are his profits for a year?
16. If a wheel turn 17° 30′ in 35 minutes, in how many hours does it make a complete revolution?

17. If a man's expenditures be $4358 in $13\frac{1}{3}$ months, what is his yearly rate of expenditure?

18. If a cistern lose by leakage 7 gals. 1 pt. in 49 hrs. 40 min., what is its hourly rate of loss?

19. If the circumference of the earth at the equator be 24,900 miles, at what rate per hour is a person there carried round, one whole rotation being made in 23 hrs. 56 min.?

20. If a man travel $3\frac{3}{5}$ miles in $7\frac{1}{2}$ minutes, how many miles will he travel in 50 minutes? and how long will he take to travel 50 miles?

If a man can do a piece of work in 9 days, in 1 day he can do $\frac{1}{9}$ of the work.

If another man can do the same work in 8 days, in 1 day he can do $\frac{1}{8}$ of it; and

Both men together can do $\frac{1}{9} + \frac{1}{8} = \frac{17}{72}$ of it in 1 day.

Therefore, if the whole work be considered to be divided into 72 equal parts, they can do 17 of these parts in 1 day, and the time required to do the whole work will be $72 \div 17 = 4\frac{4}{17}$ days.

Ex. A cistern can be filled by means of a water-pipe in 30 minutes, and can be emptied by a waste-pipe in 20 minutes. If the cistern be full, and both pipes be opened, in what time would it be emptied?

The waste-pipe empties $\frac{1}{20}$ every minute.

The water-pipe fills $\frac{1}{30}$ every minute.

When both are open $\frac{1}{20} - \frac{1}{30}$ is emptied every minute.

$$\frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60}.$$

Therefore the whole will be emptied in 60 minutes.

21. If A can mow a certain meadow in 4 days, and B in 3 days, how long will it take both?

22. If A can lay a certain wall in $4\frac{1}{2}$ days, and B in $5\frac{1}{2}$ days, how long will it take both?

23. If a pipe will fill a vessel in $4\frac{1}{2}$ hours, and another in $3\frac{1}{2}$ hours, how long will it take both to fill the vessel?

24. If A can go from Boston to Albany in $9\frac{1}{4}$ hours, and B from Albany to Boston in $11\frac{1}{3}$ hours, and they start at the same time, in how many hours will they meet?

25. A requires 4 days, B 3 days, and C $4\frac{1}{2}$ days, to do a certain piece of work. How long will it take, all three working together?

26. A can mow $\frac{5}{9}$ of a field in 3 days; B can mow $\frac{2}{3}$ of it in 4 days. How long will it take both to mow the field?

27. One pipe can fill a cistern half full in $\frac{3}{4}$ of an hour, and another can fill it three-quarters full in $\frac{1}{2}$ an hour. How long will it take both pipes to fill the cistern?

28. A cistern which holds 100 gallons can be filled from a pipe in 25 minutes, and emptied by a waste-pipe in 45 minutes. If both are opened together, how long will it take to fill the cistern, and how much water will be wasted?

29. A pipe can fill a cistern one-third full in $\frac{1}{4}$ of an hour; a waste-pipe can empty $\frac{1}{4}$ of the cistern in 20 minutes. If both pipes are opened, in what time will the cistern be filled?

30. If one pipe runs into a cistern at the rate of 2 gallons in 3 minutes, and another at the rate of 5 gallons in 4 minutes, while the water is running out of a third pipe at the rate of 4 gallons in 5 minutes, how long will it take to gain 71 gallons in the cistern?

31. A and B can do a piece of work in $2\frac{1}{2}$ days; A and C in $3\frac{1}{3}$ days; B and C in $4\frac{1}{4}$ days. Required the time in which all three, working together, can do the work, and in which each can do it alone.

Notice that by working *two days each* they can do $\frac{1}{2\frac{1}{2}} + \frac{1}{3\frac{1}{3}} + \frac{1}{4\frac{1}{4}}$ of the work.

Ex. A person spends $\frac{3}{8}$ of his money for dry goods, $\frac{7}{9}$ of the remainder for groceries, and has \$15 left. How much had he at first?

After spending $\frac{3}{8}$ of his money he had $\frac{5}{8}$ left.
After spending $\frac{7}{9}$ of $\frac{5}{8}$ of his money he had left $\frac{2}{9}$ of $\frac{5}{8} = \frac{5}{36}$.
Then, \$15 $= \frac{5}{36}$ of his money.
Therefore, the whole money $=$ \$15 $\div \frac{5}{36} =$ \$108. *Ans.*

32. Sampson & Reed sold $\frac{5}{8}$ of a lot of wheat to one party, $\frac{3}{4}$ of the remainder to another, and had 93 bushels left. How much had they at first?
33. In a certain school $\frac{9}{16}$ of the scholars are girls, $\frac{4}{7}$ of the boys are over 16 years old, and 6 boys are under 16. How many girls, and how many scholars in all?
34. In a certain school $\frac{13}{24}$ are boys; $\frac{9}{22}$ of the girls are under 16, and 13 girls are over 16. How many boys and how many girls in the school?
35. If from a certain number $\frac{3}{4}$ of it be subtracted, then $\frac{1}{5}$ of the remainder, then $\frac{1}{7}$ of that remainder, and 6 still remain, what is the number?
36. 20 is $\frac{5}{8}$ of $\frac{2}{5}$ of $\frac{2}{3}$ of what number?
37. 6 is $\frac{6}{7}$ of $\frac{4}{5}$ of $\frac{1}{4}$ of what number?
38. Express $\frac{1}{25}$ of 1 lb. troy $+ \frac{1}{25}$ of 1 lb. avoirdupois as troy and as avoirdupois weights.
39. The cargo of a ship, worth \$45,000, belongs to three partners. A owns $\frac{7}{9}$ of $\frac{3}{5}$ of it, B's share is equal to $3\frac{3}{14}$ of $\frac{2}{9}$ of A's share, and C owns the remainder. What ought each to receive from the sale?
40. Find the largest number which is contained an integral number of times in each of the following: $2\frac{5}{9}$, $6\frac{7}{18}$, $11\frac{1}{2}$, $19\frac{1}{6}$.
41. A person bequeathed $\frac{5}{12}$ of his property to A, $\frac{1}{4}$ of it to B, $\frac{1}{6}$ to C, $\frac{1}{8}$ to D, and the remainder, \$550, to E. What was the value of the whole property?
42. Arrange in descending order of magnitude, $\frac{13}{25}$, $\frac{15}{26}$, $\frac{16}{27}$.

43. A bankrupt's debts are $2520, and the value of his property is $1890. How much can he pay on a dollar?

44. A bankrupt's debts are $4264, and he pays $62\frac{1}{2}$ cents on a dollar, what are his assets?

45. If 15 yards of silk cost $18.75, how much will $20\frac{1}{3}$ yards cost?

46. If $3\frac{2}{5}$ pounds of tea cost $3.80, how much can I buy for $21.87?

47. If $\frac{3}{14}$ of a ton of coal cost $1.12, what is the price of $5\frac{1}{2}$ cwt.?

48. If $\frac{2}{11}$ of a piece of work be done in 25 days, how much will be done in $11\frac{2}{3}$ days?

49. A man walks 18 mi. 106 rds. $3\frac{2}{3}$ yards in $5\frac{1}{2}$ hours. How long does he take to walk a mile and a half?

50. When an ounce of gold is worth $19.45, what is the value of .04 of a pound?

Ex. If 27 men can do a piece of work in 14 days, working 10 hours a day, how many hours a day must 12 men work to do the same work in 45 days?

Since 27 men can do the work in 14×10 hours $= 140$ hours,
1 man can do it in 27×140 hours,
and 12 men can do it in $\frac{27 \times 140}{12}$ hours $= 315$ hours.
Hence, the number of hours they work each day
$= \frac{315}{45}$, or 7. *Ans.*

51. If 9 horses can plow 46 acres in a certain time, how many acres can 12 horses plow in the same time?

52. If 12 men can reap a field in 4 days, in what time will 32 men reap it?

53. If 72 men dig a trench in 63 days, in how many days will 42 men dig another trench three times as great?

54. If a man travel 540 miles in 24 days, walking 6 hours a day, how many miles can he travel in 3 days, walking 8 hours a day?

55. If 15 men can perform a piece of work in 22 days, how many men will finish another piece of work four times as large in $\frac{1}{5}$ of the time?

56. A garrison of 2100 has provisions for 9 months, but receives reinforcements of 600 men. How long will the provisions last?

57. If a cubic foot of ice weigh $57\frac{3}{8}$ pounds, how many cubic feet of ice will weigh a ton?

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(1) If 12 horses can plow 96 acres in 6 days, how many horses will plow 64 acres in 8 days?

In 6 days 96 acres can be plowed by 12 horses.

In 1 day 96 acres can be plowed by 6×12 horses.

In 1 day 1 acre can be plowed by $\frac{6 \times 12}{96}$ horses.

In 8 days 1 acre can be plowed by $\frac{6 \times 12}{8 \times 96}$ horses.

In 8 days 64 acres can be plowed by $\frac{64 \times 6 \times 12}{8 \times 96}$ horses.

$$\frac{\overset{8}{\cancel{64}} \times 6 \times \cancel{12}}{\cancel{8} \times \underset{8}{\cancel{96}}} \text{ horses} = 6 \text{ horses. } \textit{Ans.}$$

(2) If 40 acres of grass be mowed by 8 men in 7 days, how many acres will be mowed by 24 men in 28 days?

24 men will mow *three times* as much as 8 men in the same time.

And the same number of men will mow *four times* as much in 28 days as in 7 days.

Hence, 24 men in 28 days will mow 3×4 times as much as 8 men in 7 days.

$3 \times 4 \times 40$ acres = 480 acres. *Ans.*

58. How many bushels of wheat will serve 72 people 8 days when 4 bushels serve 6 people 24 days?

59. If 2 horses eat 8 bushels of oats in 16 days, how many horses will eat 3000 bushels in 24 days?

60. If a man travel 150 miles in 5 days, when the days are 12 hours long, in how many days of 10 hours each will he travel 500 miles?

61. If a regiment of 939 soldiers consume 351 bushels of wheat in 21 days, how many soldiers will consume 1404 bushels in 7 days?

62. If 5 men can reap a field of $12\frac{1}{2}$ acres in $3\frac{1}{2}$ days, working 16 hours a day, in what time can 7 men reap a field of 15 acres, working 12 hours a day?

63. If 7 men mow 22 acres in 8 days, working 11 hours a day, in how many days, working 10 hours a day, will 12 men mow 360 acres?

64. If 44 cannon, firing 30 rounds an hour for 3 hours a day, consume 300 barrels of powder in 5 days, how long will 400 barrels last 66 cannon, firing 40 rounds an hour for 5 hours a day?

65. How many times will a wheel $2\frac{4}{7}$ feet in circumference turn round in travelling over $12\frac{6}{7}$ yards?

66. How much ground will be travelled over by a wheel $1\frac{3}{7}$ yards in circumference, when it has made $4\frac{1}{2}$ turns?

67. Find the circumference of a wheel which makes 9 turns in travelling over $7\frac{1}{5}$ yards.

68. If the circumference of a wheel be $\frac{22}{7}$ of 1 yd. $1\frac{1}{3}$ ft., how many times will it turn in travelling $3\frac{3}{7}$ miles?

69. If the wheel of a locomotive be $3\frac{1}{7}$ times 5.52 feet in circumference, how many times does it turn in a minute, when the locomotive is running at the rate of 13.34 miles an hour?

70. A can run $\frac{4}{55}$ of a mile in $\frac{3}{5}$ of a minute, B can run $\frac{5}{56}$ of a mile in $\frac{2}{3}$ of a minute, and C $\frac{2}{19}$ of a mile in $\frac{5}{7}$ of a minute. Which is the fastest runner? and if he can run a certain distance in 3 min. 10 sec., how much longer will each of the others take to run the same distance?

71. Find the amount of the following bill:

Boston, Nov. 28, 1880.

Mr. Richard Rowe,

To John Doe, Dr.

To 125 lbs. sugar @ 10 cts.	$	
" 1 bag coffee, 115 lbs. @ 32 cts. .		
" 25 gals. molasses @ 62 cts. . .		
" 8 lbs. Japan tea @ 92 cts. . .		
" 28 lbs. crackers @ 8 cts. . . .		
" 2 bbls. flour @ $7.50		

$

Received Payment, John Doe.

Make bills for the following transactions:

72. James Hardy bought of C. H. Mills 275 bbls. of flour, at $6.75; 324 bbls. of flour, at $6.25; 300 bu. of potatoes, at 48 cents; 1578 lbs. of butter, at 32 cents; 2000 bbls. of apples, at $1.25; a car-load (20,000 lbs.) of oats, at 42 cents a bushel; a car-load (28,575 lbs.) corn, at 55 cents a bushel.

73. James Harlow bought of John Pike 12 bales, 480 lbs. each, Texas cotton, at 9¼ cents; 7 bales, 502 lbs. each, upland, at 10¼ cents; 3 bales, 492 lbs. each, low middling, at 9¾ cents; 18 bales, 490 lbs. each, good ordinary, at 9 cents.

MEASURES OF SURFACE.

It will be remembered that if the length and breadth of a rectangle be expressed in the same linear unit, the product of these two numbers will express its area in square units of the same name as the linear units of the sides.

And, conversely, the number of square units divided by the number of linear units in one side will give the number of linear units in the other side.

(1) Find the area of a floor 16 ft. 3 in. long, 12 ft. 6 in. wide.

$$16 \text{ ft. } 3 \text{ in.} = 16\tfrac{1}{4} \text{ ft.} \qquad 12 \text{ ft. } 6 \text{ in.} = 12\tfrac{1}{2} \text{ ft.}$$

$$16\tfrac{1}{4} \times 12\tfrac{1}{2} = \tfrac{65}{4} \times \tfrac{25}{2} = \tfrac{1625}{8} \text{ sq. ft.}$$

$$= 203\tfrac{1}{8} \text{ sq. ft. } \textit{Ans.}$$

(2) A rectangle contains 670 sq. ft. 108 sq. in., and is 19 ft. 6 in. wide. Find its length.

$$670 \text{ sq. ft. } 108 \text{ sq. in.} = 670\tfrac{3}{4} \text{ sq. ft.} \qquad 19 \text{ ft. } 6 \text{ in.} = 19\tfrac{1}{2} \text{ ft.}$$

$$670\tfrac{3}{4} \div 19\tfrac{1}{2} = \frac{2683}{\underset{2}{\cancel{4}}} \times \frac{\cancel{2}}{39} = \frac{2683}{78}$$

$$= 34\tfrac{31}{78} \text{ ft. } \textit{Ans.}$$

EXERCISE L.

1. What length of board 15 in. wide will contain 11 sq. ft. 36 sq. in.?
2. What length of road 44 ft. wide will contain an acre?
3. Find the area of a rectangular field 13.12 chains long, 10.35 chains broad.
4. A path 216 ft. long measured 72 sq. yds. Find its breadth.
5. A rectangular field of 21.66 acres is 250.8 yds. broad. Find its length.
6. What is the area of a table if length and breadth be 4 ft. $3\tfrac{3}{7}$ in. and 2 ft. $9\tfrac{3}{8}$ in., respectively?

7. From each corner of a square, the side of which is 2 ft. 5 in., a square measuring 5 in. on a side is cut out. Find the area of the remainder of the figure.
8. The length and breadth of a map are 4½ ft. and 3⅓ ft., respectively. If the map represent 77,760 sq. mi. of country, how many miles are there to a square inch?
9. In rolling a grass plot 24 yds. long and containing 400 sq. yds., how many times must a roller 3 ft. 4 in. wide be drawn over it lengthwise so that the whole may be rolled?
10. How many sods, each 2 ft. 3½ in. long and 8¼ in. broad, would be required to turf an acre of ground?
11. Find the area of a picture-frame 2¼ in. broad and having an outside measurement of 4 ft. 6½ in. in length and 2 ft. 8 in. in width.
12. Find the expense of glazing four windows, each containing 12 panes, the panes being each a foot long and 10 in. wide, and the price of the glass 38 cents per square foot.
13. A garden 76 yds. long and 56 yds. broad, enclosed by a wall, has a border 4 ft. wide within the wall, and within this a path 5 ft. wide, the middle being grass. Find the areas of the border, path, and grass, respectively.

It will be remembered that the area of a circle is found by multiplying the square of the radius by 3.1416.

14. Find the area of a circle which has a radius of 3 ft.
15. What is the area of a circular field with a radius of 400 yards?
16. The radius of the rotunda of the Pantheon at Rome is 71 ft. 6 in. Find the area of the floor.
17. The diameter of a cistern is 13 ft. What is the area of the bottom?

18. The two dials of the clock of St. Paul's, London, are each 18¼ feet in diameter. What is the area of each in square feet?

It will be remembered that the surface of a sphere is found by multiplying the square of the diameter by 3.1416.

19. How many square inches on the surface of a ball 3 in. in diameter?

20. How many square inches of surface in a spherical blackboard 12 in. in diameter?

21. What is the interior surface of a hemispherical vase 20 inches in diameter?

CARPETING ROOMS.

It will be remembered that in determining the number of yards of carpeting required for a room, we first decide whether the strips shall run lengthwise or across the room, and then find the number of strips needed.

The number of yards in a strip multiplied by the number of strips will give the required number of yards.

22. How many yards of carpeting ¾ of a yard wide will be required for a floor 26 ft. long, 15¾ ft. wide, if the strips run lengthwise? How many if the strips run across the room? How much will be turned under in each case?

23. How many yards ⅞ of a yard wide will be required for a room 8½ yds. long and 17 ft. wide, if the strips run lengthwise, and there is a waste of $\frac{1}{16}$ of a yard in each strip, in matching patterns?

24. How many square yards of oil-cloth will be required for a hall floor 5¼ yds. long and 10 ft. wide?

25. What will be the cost of carpet $\frac{5}{4}$ of a yard wide for a room 28½ ft. by 18¾ ft., if the strips run lengthwise, and the cost per yard is 92 cents?

26. Find the cost of carpet 30 inches wide, at $1.25 per yd. for a room 18 ft. by 14 ft., if the strips run lengthwise? if the strips run across the room?

27. Find the cost of carpeting 27 inches wide, at $1.12½ per yard, for a room 29 ft. 9 in. by 23 ft. 6 in., if the strips run across the room?

28. Find the cost of carpeting ¾ of a yard wide, at $2.75 per yard, for a room 34 ft. 8 in. by 13 ft. 3 in., if the strips run lengthwise, and if there be a waste of ¼ of a yard on each strip in matching the pattern?

29. Which way must the strips of carpet ¾ of a yard wide run in order to carpet most economically a room 20 ft. 6 in. long and 19 ft. 6 in. wide, if there be no waste for matching the pattern?

PAPERING AND PLASTERING.

It will be remembered that the area of the four walls of a room is equal to the perimeter × height.

30. Find the number of yards of plastering in the walls of a room 21¾ ft. long, 16½ ft. wide, and 11 ft. high, if 12 sq. yds. be allowed for doors, windows, and baseboards.

31. How many square yards of plastering in the walls and ceiling of a room 30 ft. 8 in. long, 26 ft. 5 in. wide, 10 ft. 6 in. high, if 24 sq. yds. be allowed for doors, windows, and base-board?

32. What will be the cost of plastering the walls and ceiling of a room 27 ft. 4 in. long, 20 ft. wide, and 12 ft. 6 in. high, at 27 cents per square yard, if 20 sq. yds. be deducted for doors, windows, and base-board?

33. Find the cost of whitening the ceiling and walls of a room 14 ft. 4 in. wide, 15 ft. 6 in. long, 10 ft. 6 in. high, at 5 cents per square yard, allowing 9 square yards *for doors* and windows.

34. Find the cost of plastering a room 21 ft. long, 15 ft. wide, 12 ft. high, at 40 cents per square yard, allowing for a door 7 ft. high, 3 ft. wide; 3 windows, each 5 ft. high, 3 ft. wide; and a dado 2 ft. 9 in. high around the room?

35. Find the cost of papering a room 20 ft. 6 in. long, 17 ft. 4 in. wide, 9 ft. high, with paper 18 in. wide, 8 yards in a roll, at 75 cents a roll; allowing for 2 doors, each 7 ft. high, 3 ft. wide, and for 3 windows, each 5 ft. 6 in. high and 3 ft. 3 in. wide.

36. Find the cost of papering a room 32 ft. long, 22 ft. wide, 13 ft. high, with paper 18 in. wide, 8 yards in a roll, at $1.25 a roll, if 50 sq. yds. be allowed for doors, windows, and base-board.

37. Find the cost of papering a room 26 ft. long, 21 ft. wide, 12 ft. high, with paper 20 in. wide, 8 yards in a roll, at $1.50 a roll, and a border at 25 cents per running foot; allowing for a fire-place 5 ft. 3 in. by 4 ft., a door 7 ft. by 4½ ft., and 3 windows, each 6 ft. by 3½ ft.

BOARD MEASURE.

318. Boards 1 inch or less in thickness are sold by the square foot.

Boards more than 1 inch in thickness, and all squared lumber, are sold by the number of square feet of boards 1 inch in thickness to which they are equivalent.

Thus, a board 16 feet long, 1 foot wide, and 1 inch thick, contains 16 feet *board measure.* If only ⅞, ¾, or ½ of an inch thick, it still contains 16 feet; but if 1¼ inches thick, it contains 20 feet board measure.

In practice, the width of a board, unless sawed to order, is reckoned only to the next smaller half-inch. Thus, a width of 11⅜ inches is reckoned 11 inches; of 13⅝ or 13¾ inches, is reckoned 13½ inches.

How many feet board measure in:

38. A board 18 ft. long, 9 in. wide, $\frac{7}{8}$ in. thick?
A board 16 ft. long, 11 in. wide, 1 in. thick?

39. Twenty boards averaging 14 ft. long, 10 in. wide, $\frac{1}{2}$ in. thick?

40. Three joists 13 ft. long, 8 in. wide, 3 in. thick?

41. A stick of timber 8 in. by 9 in. and 27 ft. long.

42. Two beams, each 6 in. by 9 in. and 23 ft. long?

43. Three joists, each 3 in. by 4 in. and 11 ft. long?

44. Five joists, each 6 in. by 4 in. and 14 ft. long?

45. A stick of timber 10 in. square and 36 ft. long?

46. Ten planks, each 13 ft. long, 15 in. wide, 2 in. thick?

Find the cost of:

47. Nine joists, each 15 ft. long, $3\frac{1}{2}$ in. by 5 in., at $12 per M.

NOTE. The abbreviation "per M." means by the thousand.

48. Thirty planks, each 12 ft. long, 11 in. wide, 3 in. thick, at $15 per M.

49. Four sticks of timber, each 8 in. by 9 in. and 23 ft. long, at $18 per M.

50. A board 24 ft. long, 23 in. wide at one end and 17 in. at the other, and $1\frac{1}{2}$ in. thick, at $30 per M.

51. A stick of timber 29 ft. long, 10 in. by 12 in., at $13.50 per M.

52. The flooring for two floors, each 23 ft. by 17 ft., each floor double, and of boards $\frac{7}{8}$ in. thick; the lower floor at $18, and the upper at $24, per M.

53. The flooring timbers for a room 23 ft. by 17 ft., at $18 per M., if they are 2 in. by 10 in., 17 ft. long, and are placed on edge, two close to the walls and the others with spaces of $\frac{39}{40}$ of a foot between them.

319. Round logs are sold by the amount of square lumber that can be cut from them. When they do not exceed

16 ft. in length, the length and the diameter of the small end (not the average diameter) are taken, and a table stamped upon calipers gives the calculated number of feet board measure. This table may be calculated by the following rule:

Express the diameter in inches; subtract twice the diameter from the square of the diameter, and $\frac{21}{40}$ of the remainder will express the number of feet board measure in a log ten feet long.

Ex. Find the number of feet board measure in a log 12 ft. long and 20 in. in diameter.

$20^2 - 2 \times 20 = 400 - 40 = 360,$
$\frac{21}{40}$ of $360 = 189,$
$\frac{12}{10}$ of $189 = 226.8$ ft. board measure.

The square of the diameter − twice the diameter = 360, and $\frac{21}{40}$ of 360 = 189, which would be the number of feet board measure if the log were 10 ft. long. As the log is 12 ft. long, it is necessary to take $\frac{12}{10}$ of 189 to obtain the number of feet in the whole log.

The rule may be remembered by the formula $\frac{21}{40}(d^2 - 2d)$, in which d is the diameter of the log in inches.

By this rule find the number of feet board measure in:

54. A log 14 ft. long, 17 in. in diameter.
55. A log 11 ft. long, 13 in. in diameter.
56. A log 16 ft. long, 20 in. in diameter.
57. A log 12 ft. long, 15 in. in diameter.

Find the value, at $9 per M. of:

58. A log 17 ft. long, averaging 11 in. in diameter.
59. A log 18 ft. long, averaging 13 in. in diameter.
60. A log 13 ft. long, 16 in. in diameter.
61. A log 14 ft. long, 12 in. in diameter.

320. Large, heavy timber is often sold by the ton, signifying 50 cubic feet, or 600 feet board measure.

321. Clapboards are usually 4 ft. long and 6 in. wide, and are nailed on to expose 3½ or 4 inches of their width to the weather.

62. How many clapboards will be required to cover the front of a house 60 ft. long and 20 ft. high, if they are laid 4 in. to the weather, and if 120 sq. ft. be deducted for doors and windows?

322. Shingles are 16 in. long, and are laid with ⅓ of their surface exposed to the weather.

63. If one thousand shingles cover 120 sq. ft. of roof, what is the average width of a shingle?

64. Allowing one thousand shingles for 120 sq. ft., how many thousand will be required to cover the pitched roof of a house 60 ft. long, if the width of each side of the roof be 24½ ft.?

MEASURES OF VOLUME.

It will be remembered that if the length, breadth, and height of a rectangular solid be expressed in the same linear unit, the product of these numbers will express its volume in cubic units of the same name as the linear unit of the edges.

EXERCISE LI.

1. Find the volume of a rectangular solid whose length, breadth, and thickness are 7 ft., 2 ft. 6 in., and 11 in. respectively.
2. How many cubic feet of air in a hall 54 ft. long, 33 ft. wide, 21 ft. 4 in. high?
3. Find the volume of a cube whose edge is 2½ yds.
4. A cellar is dug 21 ft. long, 17 ft. 3 in. wide, 9 ft. deep. How many cubic yards of earth are taken out?
5. How many cubic feet of water does a cistern hold whose length, breadth, and height are 5 ft. 4 in., 3 ft. 6 in., 2 ft. 10 in., respectively?

6. If the dimensions of a brick be 8 in. by 3½ in. by 2¼ in., find its volume.
7. In a bar of iron 21 ft. long, 3 in. wide, 2 in. thick, how many cubic inches are there?
8. What is the value of a bar of gold 8 in. long and ¾ of an inch square, at $190 a cubic inch?

If two of the dimensions of a rectangular solid be given, and also its volume, the third dimension is found by dividing the volume by the product of the two given dimensions.

9. A reservoir whose length and breadth are 15 yds. and 12 yds., respectively, holds 330 cu. yds. of water. What is its depth?
10. What length must be cut off a beam 9 in. by 15 in. to contain 2½ cu. ft.?
11. How high should a room be made, if its length be 31 ft. 3 in. and breadth 24 ft., in order that it may contain 10,000 cu. ft. of air?
12. A piece of wood 5 ft. long, 1 ft. broad, and 9 in. thick, is cut up into matches 2½ in. long and .1 of an inch square. How many will there be if no allowance be made for waste in cutting?
13. How long a wall 6 ft. high, 12¾ in. thick, could be built with the bricks forming a pile 17 ft. 6 in. long, 5 ft. wide, 4 ft. 3 in. high?

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14. Find the surface of a cube whose edge is 3 ft. 5⅞ in.
15. Find the surface of a rectangular block of stone 4 ft. long, 2½ ft. broad, 1¼ ft. thick.
16. A lake whose area is 45 A. is covered with ice 3 in. thick. Find the weight of the ice in tons, if a cubic foot weigh 920 oz. avoirdupois.
17. How many bricks will be required to build a wall 75 ft. long, 6 ft. high, and 16 in. thick, each brick being 8 in. long, 4 in. wide, 2¼ in. thick?

18. Find the cost of making a road 110 yds. in length and 18. ft wide, the soil being first removed to the depth of 1 ft. at a cost of 25 cts. a cubic yard; rubble being then laid 8 in. deep, at 25 cts. a cubic yard, and gravel placed on top 9 in. thick, at 62½ cents a cubic yard.

19. A room whose length is 27 ft., breadth 24 ft., height 10 ft., is to have its ceiling raised so as to increase the space by 84 cu. yds. What will then be its height?

20. A block of wood 5 ft. 4.8 in. long, 1 ft. 9 in. wide and thick, weighs 7.56 cwt. Determine the weight, in pounds, of a cubic foot.

It will be remembered that a cord is a pile 8 ft. long, 4 ft. wide, and 4 ft. high.

21. How many cords in a pile of wood 40 ft. long, 4 ft. wide, 5 ft. 4 in. high?

22. A pile of wood containing 67½ cords is 270 ft. long, and 4 ft. wide. How high is it?

23. What will be the cost of a pile of wood 25 ft. long, 4 ft. wide, 4 ft. 8 in. high, at $3.75 a cord?

24. What must be the length of a load of wood 3½ ft. high and 5 ft. wide, to contain a cord?

25. How high must manure be in a cart 6 ft. by 4 ft., in order to be ½ a cord?

CAPACITY OF BINS AND CISTERNS.

It will be remembered that the standard bushel of the United States contains 2150.42 cubic inches; and the standard liquid gallon 231 cubic inches.

Ex. Find the number of bushels in a bin that is 6 ft. long, 5 ft. wide, 4 ft. deep.

$6 \times 5 \times 4 = 120;$

120 cu. ft. $= 120 \times 1728$ cu. in. $= 207{,}360$ cu. in.

207,360 cu. in. $\div$ 2150.42 cu. in. $= 96.4.$

Therefore, the bin will hold 96.4 bu.

NOTE. Since a bushel contains 2150.42 cu. in., and a cubic foot 1728 cu. in., a bushel contains $1\frac{1}{4}$ cu. ft., nearly. Hence, practically, *any number of cubic feet diminished by $\frac{1}{5}$ of itself will represent an equivalent number of bushels.*

And *any number of bushels increased by $\frac{1}{4}$ of itself will represent an equivalent number of cubic feet.*

Thus, if a bin contain 120 cu. ft., $120 - \frac{1}{5}$ of $120 = 96$, the number of bushels the bin will hold.

And, if a bin will hold 96 bu., $96 + \frac{1}{4}$ of $96 = 120$, the number of cubic feet in the bin.

26. Find the number of bushels in a bin that is 8 ft. long, 4 ft. wide, 3 ft. deep.

27. Find the number of bushels in a bin 9 ft. long, 6 ft. 6 in. wide, 3 ft. 4 in. deep.

28. Find the depth of a bin to hold 360 bu., if its length be 12 ft. and its width 6 ft.

29. Find the length of a bin that is 6 ft. wide and 5 ft. deep, if it hold 400 bu.

30. Find the number of bushels that will fill a bin 8.5 ft. long, 4.5 ft. wide, 3.5 ft. deep.

31. A bin 20 ft. long, 12 ft. wide, and 6 ft. deep, is full of wheat. What is its value, at $1.25 a bushel?

32. If a ton of coal occupy 40 cu. ft., how many tons will a bin hold that is 21 ft. long, 10 ft. wide, 5 ft. deep?

33. If a ton of Lehigh coal occupy 35 cu. ft., how many tons will a bin hold that is 8 ft. long, 5 ft. 9 in. wide, 4 ft. 6 in. deep?

Ex. How many gallons will a cistern hold that is 5 ft. square and 6 ft. deep?

$5 \times 5 \times 6 = 150$;
150 cu. ft. $= 150 \times 1728$ cu. in. $= 259{,}200$ cu. in.
259,200 cu. in. $\div$ 231 cu. in. $= 1122\frac{6}{77}$.
Therefore, the cistern will hold $1122\frac{6}{77}$ gals.

34. Find the number of gallons that a cistern will hold that is 13 ft. long, 6 ft. wide, 7 ft. 4 in. deep.

35. Find the number of gallons that a tank will hold that is 4 ft. long, 2 ft. 8 in. wide, 1 ft. 8 in. deep.

36. Find the number of gallons in a cubic foot.

NOTE. In practice, 7½ gallons are allowed for a cubic foot.

37. Find the capacity of a cistern, in cubic feet, that will hold 200 barrels of water.

It is to be observed that the capacity of a round cistern is found by multiplying the area of the base by the depth.

38. Find the number of gallons that a round cistern will hold that is 6 ft. in diameter and 7 feet deep.

39. Find the number of gallons that a vessel will hold that is 12 in. in diameter and 10 in. deep?

40. How many quarts will a round vessel hold 5¼ in. in diameter and 6 in. deep?

It will be remembered that the capacity of a sphere is found by multiplying the cube of the diameter by .5236 (⅙ of 3.1416). § 211.

41. Find the number of cubic inches in a sphere 11 in. in diameter.

42. How many quarts will a sphere hold that is 12 in. in diameter?

43. What part of a bushel will a hemispherical bowl hold that is 13 in. in diameter?

44. If a cubical box 2 ft. on an edge contain a solid sphere 2 ft. in diameter, how many gallons of water can be poured into the box?

45. If 64 qts. of water be poured into a vessel that will hold 2 bu. of wheat, what part of the vessel will be filled?

CHAPTER XIV.

PROPORTION.

323. The *relative magnitude* of two numbers is called their **ratio**, and is expressed by the fraction which the first is of the second.

324. The terms of this fraction are called the **terms** of the ratio; the numerator is called the **antecedent**; the denominator is called the **consequent**.

Thus, the ratio of 2 to 3 is expressed by $\frac{2}{3}$, of which the numerator 2 is the antecedent, and the denominator 3 is the consequent.

325. The ratio $\frac{2}{3}$ is often written, 2:3.

326. If both terms of a ratio be multiplied or divided by the same number, the ratio is not altered.

Thus, if both terms of the ratio 2:3 be multiplied by 4, the resulting ratio is 8:12, and the ratio 8:12 is equal to the ratio 2:3; for, $\frac{8}{12} = \frac{2}{3}$. Again, if the ratio $2\frac{1}{2} : 3\frac{1}{3}$ be multiplied by 6, the resulting ratio is 15:20, and the ratio $2\frac{1}{2} : 3\frac{1}{3}$ is equal to 15:20; for, $\frac{2\frac{1}{2}}{3\frac{1}{3}} = \frac{15}{20}$. Since $\frac{15}{20}$ reduced to its lowest terms $= \frac{3}{4}$, the simplest expression for the ratio of $2\frac{1}{2} : 3\frac{1}{3}$ is 3:4.

327. If the numerator and denominator of a fraction be interchanged, the fraction is said to be *inverted;* likewise, if the antecedent and consequent of a ratio be interchanged, the resulting ratio is called the *inverse* of the given ratio.

Thus, if the fraction $\frac{3}{4}$ be inverted the resulting fraction is $\frac{4}{3}$; and the inverse of the ratio 3:4 is 4:3.

328. If two *quantities* be expressed in the *same unit*, their ratio will be the same as the ratio of the two *numbers* by which they are expressed.

Thus, the quantity \$5 is the same fraction of \$11 as 5 is of 11.

329. Since ratio is simply *relative magnitude*, two quantities *different in kind* cannot form the terms of a ratio.

330. Two quantities the same in kind must be expressed in a *common unit* before they can form the terms of a ratio.

Thus, no ratio exists between 5 *tons* and 30 *days;* and the ratio of 5 tons to 3000 pounds can be expressed only when *both* quantities are written as tons or pounds.

331. Ratios are mere numbers, and may be compared.

Ex. Which is the greater ratio, 5 : 7 or 12 : 18?

$5:7 = \frac{5}{7}$, and $12:18 = \frac{12}{18} = \frac{2}{3}$.
Now $\frac{5}{7} = \frac{15}{21}$, and $\frac{2}{3} = \frac{14}{21}$.
As $\frac{15}{21}$ is greater than $\frac{14}{21}$,
the ratio 5 : 7 is greater than the ratio 12 : 18.

EXERCISE LII.

Which is the greater ratio:

1. 5 : 8 or 6 : 9?
2. 7 : 10 or 9 : 12?
3. 8 : 9 or 10 : 12?
4. 6 : 12 or 8 : 14?
5. 10 cwt. : 15 cwt. or \$7 : \$9?
6. 5 dys. : 7 dys. or 8 ft. : 11 ft.?
7. 9 yds. : 6 yds. or 5 : 3?
8. $\frac{3}{8}$ lb. : $\frac{1}{2}$ lb. or $\frac{5}{8}$ yd. : $\frac{3}{8}$ yd.?

332. When two ratios are equal the four terms are said to be in proportion, and are called **proportionals.**

Thus, 5, 3, 15, 9 are proportionals; for 5 : 3 = 15 : 9, since each of these ratios is represented by the fraction $\frac{5}{3}$.

333. A proportion is also written by putting a double colon between the ratios. Thus, 5 : 3 = 15 : 9 (read 5 to 3 = 15 to 9), may be written 5 : 3 : : 15 : 9 (read 5 is to 3 as 15 is to 9).

334. The *first* and *last* terms of a proportion are called the **extremes,** and the two *middle* terms are called the **means.**

335. *Test of a proportion.* When four numbers are proportionals, the product of the extremes is equal to the product of the means.

This is seen to be true by expressing the ratios in the form of fractions, and multiplying both by the product of the denominators.

Thus, the proportion $5:3::15:9$ may be written $\frac{5}{3}=\frac{15}{9}$; and if both be multiplied by 3×9, the result will be $5\times 9=3\times 15$.

336. Either extreme, therefore, will be equal to the product of the means divided by the other extreme; and either mean will be equal to the product of the extremes divided by the other mean. Hence, if three terms of a proportion be given, the fourth may be found.

(1) What number is to 4 as 3 is to 6?

This may be written $\frac{\text{What number}}{4}=\frac{3}{6}$?

Multiply both sides of the equation by 4.

The result is, $\text{What number}=\frac{4\times 3}{6}$?

Answer, 2.

(2) 20 is to 24 as what number is to 30?

This may be written $\frac{20}{24}=\frac{\text{What number}}{30}$?

Multiply by 30, $\frac{20\times 30}{24}=\text{What number}$?

Answer, 25.

(3) 18 is to 32 as 45 is to what number?

This may be written $\frac{18}{32}=\frac{45}{\text{What number}}$?

As these fractions are equal, their reciprocals are equal;

that is, $\frac{32}{18}=\frac{\text{What number}}{45}$?

Multiply by 45, $\frac{32\times 45}{18}=\text{What number}$?

Answer, 80.

337. When three terms of a proportion are given, the method of finding the fourth term is called the **Rule of Three.**

It is usual to arrange the quantities (that is, to *state* the question), so that the quantity required for the answer may be the fourth term. Hence, the quantity which *corresponds* to that of the required answer must be the third term.

(1) If 5 tons of hay cost \$87.50, what will 21 tons cost?

Since the *cost* of 21 tons is required, \$87.50 is the third term.

Since 21 tons will cost *more* than 5 tons, 21 tons is the second term and 5 tons the first term.

That is, 5 tons : 21 tons : : \$87.50 : What quantity?

A difficulty presents itself here, inasmuch as no meaning can be given to the product of the means (\$87.50 multiplied by 21 tons). Since, however, the ratio of 5 tons : 21 tons = the ratio of 5 : 21, the ratio 5 : 21 may be substituted for 5 tons : 21 tons.

Then, 5 : 21 : : \$87.50 : What quantity?

That is, What quantity $= \dfrac{21 \times \$87.50}{5}$? *Answer*, \$367.50.

(2) When a post 11.5 ft. high casts a shadow on level ground 17.4 ft. long, a neighboring steeple casts a shadow 63.7 yds. long. How high is the steeple?

Height is required; the height 11.5 ft. is therefore the third term.

Since the *shadow* of the steeple is the longer, the *height* of the steeple must be the greater; therefore, the second term must be the greater of the two remaining quantities expressed in the same unit. 63.7 yds. = 191.1 ft.

	Shadow.		Shadow.		Height.		Height.
	17.4 ft.	:	191.1 ft.	: :	11.5 ft.	:	What?
or,	17.4	:	191.1	: :	11.5 ft.	:	What?

That is, height of steeple $= \dfrac{191.1 \times 11.5 \text{ ft.}}{17.4} = 126.3$ ft. *Ans.*

(3) If $\frac{2}{5}$ of a peck of oats weigh $\frac{23}{7}$ of a pound, what will $\frac{3}{7}$ of a bushel weigh?

$\frac{2}{5}$ of a peck = $\frac{2}{5}$ of $\frac{1}{4}$ bu. = $\frac{1}{10}$ bu.; $\frac{1}{10} : \frac{3}{7} : : \frac{23}{7}$ lbs. : What?

That is, the weight of $\frac{8}{7}$ bu. $= \frac{8}{7} \times \frac{23}{7}$ lbs. $\div \frac{1}{10}$
$= \frac{8}{7} \times \frac{10}{1} \times \frac{23}{7}$ lbs. $= 37\frac{27}{49}$ lbs.

The beginner will find it of advantage to state the problem thus:
$\frac{1}{10}$ of a bushel *weighs* $\frac{23}{7}$ lbs.
Will $\frac{8}{7}$ of a bushel weigh *more* than $\frac{1}{10}$ bu. or *less?*

If the answer to the question be *more*, the larger number is to be the second term; if *less*, the smaller number is to be the second term, and the remaining number to be the first term.

Hence, in solving problems by the Rule of Three,

338. Make that quantity which is of the same kind as the required answer the third term.

The *numbers* by which the other two quantities are expressed, when expressed *in a common unit*, will be the first and second terms.

If, from the nature of the question, the answer will be *greater* than the third term, make the *greater* of these two numbers the *second* term; if *less*, make the *less* the *second* term, and the remaining number the first term.

Divide the product of the second and third terms by the first term, and the quotient will be the answer required.

EXERCISE LIII.

1. If 24 men can finish some work in 14 days, how long will it take 21 men to do it?
2. A well is dug in 13 days of 9 hours each. How many days of 10 hours each would it have taken?
3. A man who steps 2 ft. 5 in. takes 2480 steps in walking a certain distance. How many steps of 2 ft. 7 in. will be required for the same distance?
4. If $\frac{5}{18}$ ton cost $6, what will $7\frac{5}{9}$ cwt. cost, at the same rate?
5. If 42 yds. of carpet 2 ft. 3 in. wide are required for a room, how many yards 2 ft. 4 in. wide will be required?

6. A court was paved with 950 stones, each $1\frac{3}{8}$ sq. ft., and is re-paved with 836 stones of a uniform size. Find the size of each.

7. If a train, at the rate of $\frac{5}{18}$ of a mile per minute, take $3\frac{1}{4}$ hours to reach a station, how long will it take at the rate of $\frac{7}{15}$ of a mile a minute?

8. If a post 4 ft. 8 in. high cast a shadow 7 ft. 3 in. long, how long a shadow will a post 11 ft. high cast?

9. When a shadow 8 ft. 5 in. long is cast by a post 5 ft. 7 in. high, how high is a steeple that casts a shadow of 211 ft. at the same time?

10. If 4 men can mow a certain field in 10 hours, how many men will it take to mow it in 5 hours?

11. A tap discharging 4 gals. a minute empties a cistern in 3 hours. How long will it take a tap discharging 7 gals. a minute to empty it?

12. A pipe discharging 3 gals. 1 pt. a minute fills a tub in 4 min. 20 sec. How long will it take a pipe discharging 83 qts. a minute to fill it?

13. If both pipes of Ex. 12 discharge at the same time into the tub, how long will it take to fill it?

14. How long will it take to fill a cistern of 165 gals. by a pipe that fills one of 120 gals. in 7 min. 16 sec.?

15. A ship has sailed 1800 mi. in a fortnight. How long, at the same rate, will it take for a voyage of 5000 mi.?

16. The wheels of a carriage are 6 ft. 9 in. and 9 ft. 6 in. in circumference. How many times will the larger turn while the smaller turns 3762 times?

17. If $\frac{8}{25}$ of a ship be worth $2167, what is the value of $\frac{7}{17}$ of it?

18. What will be the weight of 18 cu. ft. 432 cu. in. of stone of which 10 cu. ft. 864 cu. in. weigh 14 cwt. 7 lbs.?

19. If 280 lbs. of flour make 360 lbs. of bread, how many four-pound loaves can be made from 1 cwt. of flour?

20. If a column of mercury 27.93 in. high weigh .76 of a pound, what will be the weight of a column of the same diameter 29.4 in. high?

21. How many francs will pay a bill of £100, when £42 10*s.* 8*d.* is equivalent to 1090.98 francs?

22. What will be the weight of a cube whose edge is 2 ft. 2 in., when a cube of the same material whose edge is 1 ft. 4 in. weighs 537.6 lbs.?

23. If a square field measuring 50 yds. 10$\frac{2}{7}$ in. on each side be worth \$2710$\frac{10}{17}$, what is the value of a square field 62 yds. 1 ft. each way?

24. A gains 4 yds. on B in running 30 yds. How much will he gain while B is running 97$\frac{1}{2}$ yds.?

25. If 10 cu. in. of gold weigh as much as 193 cu. in. of water, what is the size of a nugget weighing as much as a cubic foot of water?

26. If a garrison of 1500 men have provisions for 13 months, how long will the provisions last if it be increased by 700 men?

27. If a tree 38 ft. high be represented by a drawing 1$\frac{1}{2}$ in. high, what, on the same scale, will represent the height of a house 45 ft. high?

28. If a country 630 mi. long be represented on a raised map by a length of 5$\frac{1}{4}$ ft., by what height ought a mountain of 15,750 ft. be represented on the map?

29. A train travels $\frac{1}{4}$ of a mile in 18 sec. How many miles an hour does it travel?

30. If 4$\frac{1}{2}$ tons of coal fill a bin 9 ft. long, 5 ft. broad, 5 ft. high, how many cubic feet will be required for the coal of a steamer carrying 3 weeks' consumption at 20 tons a day?

31. If 2 lbs. of rosin be melted with 5 oz. of mutton tallow, to make a grafting wax, how many ounces of tallow will 20 oz. of the wax contain?

Compound Proportion.

339. A ratio is said to be *compounded* of two or more given ratios, when it is expressed by a fraction which is the product of the fractions representing the given ratios.

Thus, the ratios 2 : 3 and 7 : 11 are represented by the fractions $\frac{2}{3}$ and $\frac{7}{11}$; and the ratio 14 : 33, which is represented by $\frac{14}{33}$ (the product of $\frac{2}{3}$ and $\frac{7}{11}$), is said to be compounded of the ratios 2 : 3 and 7 : 11.

340. A proportion which has one of its ratios a compound ratio is called a **compound proportion.**

In stating problems in compound proportion the quantity which corresponds to the answer required is made the third term. Each *pair* of the remaining quantities is then considered *separately* with reference to the answer required. The process will be understood by the following example:

Ex. If 4 men mow 15 acres in 5 days of 14 hours, in how many days of 13 hours can 7 men mow 19½ acres?

As the answer is to be in days, make 5 days the third term.

I. *Will* it require *more* days for 7 men to mow 15 acres than it *did* for 4 men? Evidently, less.

Therefore, make 7 the first term and 4 the second.

II. *Will* it require *more* days for the same number of men to mow 19½ acres than it *did* to mow 15 acres? Evidently, more.

Therefore, make 15 the first term and 19½ the second.

III. *Will* it require *more* days of 13 hours for the same number of men to mow the same number of acres than it *did* of 14 hours? Evidently, more.

Therefore, make 13 the first term and 14 the second.

Hence, the statement is:

$$\begin{array}{r|l}7 & 4\\ 15 & 19.5 :: 5 \text{ days} : \text{what?}\\ 13 & 14\end{array}$$

As the fourth term is obtained by dividing the product of the numbers on the right of the vertical line by the product of the numbers on the left, the work may be shortened by cancelling. Thus:

$$\begin{array}{rrr|lll} & & \cancel{7} & 4 & & 1\text{ day} \\ & & \cancel{15} & \cancel{19.5} & \cancel{1.3} :: & \cancel{5}\text{ days : what?} \\ \cancel{5} & \cancel{10} & \cancel{13} & \cancel{14} & \cancel{2} & \end{array}$$

Cancel the factor 7, in 7 and 14, and set the 2 at the right of 14.

Cancel the factor 15, in 15 and 19.5.

Cancel the factor 1.3, in 1.3 and 13.

Cancel the factor 2, in 2 and 10.

Cancel the factor 5, in 5 and in 5 days, which leaves 1 day to be multiplied by 4.

Therefore, the answer is 4 days.

EXERCISE LIV.

1. How many days 8 hours long will 60 men take to finish some work which 24 men can do in 15 days, working 10 hours a day?
2. What will be the expense of covering a room with drugget 4 ft. wide, at 91⅔ cts. a yard, when carpet 2 ft. 3 in. wide for the room costs $70.50, at $1.37½ a yard?
3. If 4418 tons of iron ore produce $36,190 worth of metal, when iron is at $37.50 a ton, what will be the value of the iron from 2275 tons of ore, at $47 a ton?
4. If a bar of iron 3⅛ ft. long, 3 in. wide, 2¾ in. thick weigh 93 lbs., what will be the weight of a bar 3⅞ ft. long, 4 in. wide, and 2½ in. thick?
5. If 40 bu. of wheat can be grown on the same area as 48 bu. of barley, and 28 acres produce 840 bu. of wheat, how much barley will be obtained from 38 acres?
6. If 18 men can dig a trench 150 ft. long, 6 ft. broad, and 4 ft. 6 in. deep in 12 days, how long will 16 men take for a trench 210 ft. long, 5 ft. broad, and 4 ft. deep?
7. In the reprint of a book consisting of 810 pages, 50 lines are contained in a page, instead of 40, and 72 letters in a line instead of 60. Of how many pages will the new edition consist?

8. If 3280 42-lb. shot cost $3000, how many 32-lb. shot can be bought for $4200?

9. What must be the rate of wages, that 12 men may earn in 10 days the same amount that 9 men earn in 14 days, at $1.50 a day?

10. A reservoir 15 yds. long and 4 ft. deep holds 32,500 gals. Determine the quantity of water it will hold when it has been increased in length by 18 ft. and in depth 1 ft.?

11. How far can A, who takes 3.1 ft. each step, run, while B, who takes 2.3 ft. each step, runs 220 yds., if A takes 7 steps while B takes 11?

12. If 6 hours be required for travelling a given distance at a given rate, how long will be required when the distance is diminished by one-fourth and the rate is increased by one-half?

13. How many hours a day must 5 men work to mow the same quantity of grass in 8 days that 7 men can mow in 6 days, working 10 hours a day?

14. If a bar 10 ft. 6½ in. long. 3⅝ in. broad, 3½ in. thick weigh 4 cwt. 8.23 lbs., what length must be taken to weigh a long ton when the breadth and thickness are 4⅔ in. and 4⅛ in. respectively?

15. If 27 men, in 28 days of 10 hours each, dig a trench 126 yds. long, 2½ yds. broad, 1½ yds. deep, how long a trench 2¾ yds. broad, 1¾ yds. deep, will 56 men dig in 25 days of 8¼ hours?

16. What must be the length of a bar of silver ¾ in. square, that it may weigh the same as a bar of gold ½ in. square and 6¾ in. long, if the weight of a cubic inch of silver have to that of a cubic inch of gold the ratio 47 : 88?

17. If it take 34^{kg} of wool to make 25^{m} of cloth $.6^{m}$ wide, how long a piece of cloth $.8^{m}$ wide can be made from 108.8^{kg} of wool?

18. An oak beam 5.40^{m} long, $.63^{m}$ thick, and $.57^{m}$ wide weighs 1469.25^{kg}, find the weight of a beam whose dimensions are 4.87^{m}, $.58^{m}$, $.53^{m}$.

NOTE. The "coefficient of expansion" of a body is the increase of the unit of volume of the body when the temperature is increased one degree. For air, this coefficient is .00367. That is, a cubic foot of air at zero will occupy at 1° C. a volume equal to 1.00367 cu. ft. if allowed to expand, the pressure remaining the same.

19. A certain quantity of air has a volume of 195.5 cu. ft. at 27.8°. What will be its volume at 100°?

NOTE. The volumes occupied by the same quantity of air, the temperature remaining unchanged, are in inverse ratio to the pressures.

20. A quantity of air at a temperature of 15.6° C. has a volume of 4 cu. ft. under a pressure of 12 lbs. to the square inch. What will be its volume at a temperature of 48.7° C., and under a pressure of 14 lbs. to the square inch?

Proportional Parts.

341. If it be required to divide a quantity into parts proportional to 3, 4, 5; the *numbers* 3, 4, 5 may be taken as *representatives* of the parts, and then the whole quantity will be represented by $3 + 4 + 5$; that is, by 12.

(1) Divide \$391 into parts proportional to the numbers 5, 7, and 11.

The whole quantity will be represented by $5 + 7 + 11 = 23$.
Therefore, the respective parts will be, $\frac{5}{23}$, $\frac{7}{23}$, $\frac{11}{23}$ of \$391.
That is, \$85, \$119, \$187. *Ans.*
Or, the parts of the quantity may be found by the proportions:

$$23 : 5 :: \$391 : ?$$
$$23 : 7 :: \$391 : ?$$
$$23 : 11 :: \$391 : ?$$

In which the required terms will be \$85, \$119, and \$187 respectively.

(2) Divide $248 into parts proportional to the fractions $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{25}$.

Multiply the fractions by 150, the L.C.M. of their denominators. The results are 15, 10, 6. Hence, the parts will be *represented* by the numbers 15, 10, 6, and the whole by 31.

Therefore, the respective parts will be $\frac{15}{31}$, $\frac{10}{31}$, $\frac{6}{31}$ of $248.

That is, $120, $80, $48. *Ans.*

EXERCISE LV.

1. Divide $12,000 proportionally to the numbers 3; 4, 5.
2. Divide 815 tons proportionally to $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.
3. Divide 6853 lbs. of wool proportionally to $1\frac{3}{4}$, $2\frac{4}{5}$, $5\frac{5}{6}$; and also proportionally to the reciprocals of these numbers.
4. Two persons join in purchasing some property, one paying $1250 and the other $1000. If the property rise in value to $3600, what will be the value of each one's share?
5. Gun-metal is composed of 3 parts (by weight) of tin to 100 parts of copper. What weight of each of these metals will there be in cannon weighing 721 lbs.?
6. Bell-metal contains 78 parts copper and 22 parts tin. What weight of each of these metals will there be in a bell weighing 937 lbs.?
7. It takes 75^{kg} of saltpeter, 12.5^{kg} of charcoal, and 12.5^{kg} of sulphur to make 100^{kg} of powder. How much of each of these substances will be required to make 10,000,000 cartridges, each containing 5^{g} of powder?
8. Yellow copper contains 2 parts of red copper and 1 part zinc. How many ounces of red copper are there in an article weighing 1 lb. made of yellow copper?
9. Type-metal is made of an alloy containing 39 parts of lead to 11 parts antimony. How many pounds of each will be required to make 957 lbs. of type?

10. Plumber's solder contains 2 parts lead and 1 part tin. How much of each of these in 100 lbs. of solder?

11. The air is composed of oxygen and nitrogen. In 100 volumes of air there are 21 volumes of oxygen and 79 of nitrogen. Reckoning the weight of a liter of oxygen to be 1.4295^{g}, that of a liter of nitrogen 1.2577^{g}, find the number of grams of each gas in 100^{g} of air.

12. What is the value of the gold in a chain weighing 3 oz. 4 dwt., supposing it to be 18 carats fine (that is, 18 parts of pure gold out of 24), at $19 an ounce?

PARTNERSHIP.

342. Partnership is separated into *simple* and *compound*. In simple partnership the capital of each partner is invested for *the same time*. In compound partnership the time for which the capital of each partner is invested is taken into account, as well as the amount of the capital; and the division of profits and losses is made proportionally to the amount of the capital and the time it is invested.

EXERCISE LVI.

1. Arnold and Baker enter into partnership. Arnold puts in $6000 for 8 months, and Baker $4000 for 6 months. Their profits are $2000. What is each man's share?

NOTE. Since the use of $6000 for 8 months is equivalent to the use of $48,000 for 1 month; and the use of $4000 for 6 months is equivalent to the use of $24,000 for 1 month, their profits must be divided in the ratio 48,000 : 24,000.

2. Dobson furnishes the firm of Dobson & Fogg with $5000 for 13 months; Fogg furnishes $7000 for 9 months. Their profits are $1700. What is the share of each?

3. In a business speculation, A furnishes $800, and after 3 months $250 more; B furnishes $950, and at the

end of 2 months withdraws $200; C furnishes $650, and at the end of 6 months $400 more. At the end of a year they realize a profit of $2516. How shall it be divided among them?

4. Two partners, A and B, begin business with capitals of $3500 and $8700, and A is to have .12 of the profits for managing the business. How shall a profit of $1906.25 be divided between them?

5. A puts $2100 into a business, and B $1750. At the end of a year each puts in $700 more, and C joins them with $2500. At the end of 18 months from this time how shall a profit of $2166.50 be divided?

6. Three graziers hire a pasture, for which they pay $132.50. One puts in 10 oxen for 3 months, another 12 oxen for 4 months, and the third 14 oxen for 2 months. How much of the rent ought each to pay?

7. A begins business, with a capital of $2400, on the 19th of March; and on the 17th of July admits B as a partner, with a capital of $1800. Dec. 31 the profits are $943. What is the share of each?

8. A and B join capitals in the ratio 7 : 11. At the end of 7 months A withdraws $\frac{1}{2}$ of his, and B $\frac{1}{3}$ of his; and, after 11 months more, they divide a profit of $5148.50. What is the share of each?

9. Divide £65 9*s*. among three persons, so that the first may have as many half-crowns as the second has shillings; and the second as many guineas as the third has pounds.

10. Two partners begin business each with a capital of $2000. A adds $500 at the end of 2 months, and $500 more at the end of 7 months; B adds $800 at the end of 3 months. What is the share of each, at the year's end, of a profit of $3605.25?

CHAPTER XV.

PERCENTAGE.

343. In considering the increase or decrease in quantities, it is usual to employ, as a **common standard of reference, the number 100.**

Thus, if the population of a town at one census were 1200, and at the next 1500, the increase would be 300 in 1200; that is, 25 in every 100; or, as it is generally expressed, 25 *per cent.*

344. The symbol % is used for the words **per cent.**

345. The **representative number** resulting after an increase has taken place will be 100 + increase per cent; and after a decrease has taken place will be 100 − decrease per cent.

346. The *representative numbers* in any particular case may be changed to **quantities** by applying them all to the same unit of quantity.

Thus, if gunpowder be said to contain 75% of saltpetre, the meaning is, that if the number 100 be taken as the representative of the whole weight, the number 75 will represent the weight of saltpetre in it; and if the numbers be applied to any unit of weight, as a pound, the meaning will be, that 100 lbs. of gunpowder will contain 75 lbs. of saltpetre.

(1) Ten years ago the population of a city was 26,275, and has increased 20%. What is its present population?

If 100 be taken to represent the population ten years ago, 100 + 20 will represent the present population.

Therefore, the present population will be $\frac{120}{100}$ of 26,275 = 31,530. *Ans.*

(2) Ten years ago the population of a city was 26,275; its present population is 31,530. Determine the increase per cent.

31,530 – 26,275 = 5255, actual increase.

Since the increase on 26,275 is 5255, the increase on 100 is $\frac{100}{26275}$ of 5255 = 20.

The increase, therefore, is 20%. *Ans.*

(3) A town, after decreasing 11%, has 4539 inhabitants. Find its number at first.

If 100 be taken to represent the population at first, 100 – 11 = 89, will represent the present population.

Therefore, the population at first was $\frac{100}{89}$ of 4539 = 5100. *Ans.*

(4) In a certain school there are 200 girls, and the girls are 40% of the whole number of pupils. How many pupils in the school?

If 100 be taken to represent the whole number of pupils, 40 will represent the number of girls.

Therefore, the whole number of pupils is $\frac{100}{40}$ of 200 = 500. *Ans.*

(5) 50 lbs. is what per cent of 450 lbs.?

If 100 be taken to represent the whole weight, the number required to represent 50 lbs. will be $\frac{50}{450}$ of 100 = $11\frac{1}{9}$.

That is, $11\frac{1}{9}$%. *Ans.*

347. In the process of computing by the hundred, it is generally more convenient to use 1 as the representative number, and to express the per cent as **hundredths.**

Thus, in example (1), if the number 1 be taken to represent 26,275 inhabitants, 1.20 will represent the number of inhabitants after an increase of 20%; and the present population will be 1.20 of 26,275 = 31,530. *Ans.*

In example (2), if the number 1 be taken to represent 26,275, the increase will be represented by $5255 \div 26,275 = .20$. *Ans.*

In example (3), if 1 be taken to represent the population at first, .89 will represent its present population. That is, 4539 is .89 of the former population. Therefore, the former population was $4539 \div .89 = 5100$.

In example (4), if the number 1 be taken to represent the whole number of pupils, 200 will represent .40 of the whole number. Therefore, the whole number will be $200 \div .40 = 500$. *Ans.*

In example (5), if the number 1 be taken to represent 450 lbs., 50 lbs. will be represented by $\frac{50}{450}$ of $1 = .11\frac{1}{9}$. *Ans.*

EXERCISE LVII.

1. The population of a town in 1870 was 12,275, and it increased 8% in the next ten years. Find its population in 1880.
2. How much metal will be obtained from 365 tons of ore, if the metal be 7% of the ore?
3. If gunpowder contains 75% of saltpetre, 10% of sulphur, 15% of charcoal, how much of each is there in a ton of powder?
4. A manufactory uses 24 tons of coal a day, and 20% of it is lost in smoke. How much coal would be needed if this waste could be prevented?
5. Air consists of 20.0265% (by measure) of oxygen gas and 79.9735% of nitrogen. How much oxygen in 1750 cu. ft. of air?
6. A town, after decreasing 25%, has 4539 inhabitants. Find its number at first.
7. 2% of a regiment of 750 men are killed in an engagement, 6% are wounded, and 4% are missing. What is the number still available for service?
8. If $3\frac{3}{4}$ tons of sulphur are required to make $31\frac{1}{4}$ tons of gunpowder, what is the per cent of sulphur in gunpowder?

9. In a school of 80 children, 17½% are girls. Find the number of boys.
10. If goods are bought for $415, and sold for $500, what is the gain per cent?
11. If goods are bought for $415, and sold for $400, what is the loss per cent?
12. $500 is 4% of what number?
13. A farmer buys 24 head of cattle at $80 a head, and, after losing 6, sells the remainder at $105 a head. How much does he gain or lose per cent?
14. If a ton (2240 lbs.) of ore in a gold mine yields 5 oz. (troy) of gold, what is the yield per cent?
15. If the ore in a mine yields $\frac{3}{80}$ of 1% of pure gold, how many tons (2240 lbs.) of ore must be taken to obtain 7 lbs. (troy) of gold?
16. 12½ tons of iron are obtained from 235 tons of ore. What per cent of the ore is iron?
17. Goods are sold, at a loss of 3%, for $2667.50. What was the cost?
18. Teas at 68 cents, 86 cents, and 96 cents a pound, are mixed in equal quantities, and sold at 90 cents a pound. Find the gain per cent.
19. By selling goods for $1173.92, a merchant gains $153.12. Find the gain per cent on the cost.
20. If to 25 gals. of alcohol 2 gals. of water are added, how much per cent of the mixture is water? how much per cent is alcohol?
21. What was the cost when 17½% was gained by selling goods for $253.80?
22. A wine merchant mixes 24 gallons, at $7 a gallon, with 18 gallons, at $5 a gallon, and sells the whole at $7 a gallon. What does he gain per cent?
23. By selling a horse for $200, a dealer loses 12½%. What would he have gained or lost per cent by *selling at $250?*

24. A spirit merchant buys 75 gals., at $3.25 a gallon, and, after drawing off 10 gals., sells the remainder so as to gain 5% on the whole. What is the selling price per gallon?

25. A person owns two estates worth respectively $9845 and $12,155. If the first rise in value 32%, and the second fall 13%, determine the rise or fall per cent in the value of his whole property.

26. A tradesman marks an article $5, but takes off 5% for cash. If his profit is 14%, what was the cost of the article?

27. What would a dishonest dealer gain per cent by using a false weight of 15 oz. instead of a pound?

28. A dishonest dealer gains 12% by using false weights. What is the real weight of his pound?

29. A tradesman, in selling goods, deducts from the marked price 5% for cash. What is the marked price of some goods for which he receives $7.12½?

30. The lead ore from a certain mine yields 60% of metal, and of the metal ¾ of 1% is silver. How much silver and lead will be obtained from 1200 tons of ore?

31. If ore loses 41½% of its weight in roasting, and 43¾% of the remainder in smelting, how much ore will be required to yield 1000 tons of metal?

Ex. How many per cent above cost must a man mark his goods in order that he may take off 20% from the marked price, and still make 20% on the cost?

If 100 be taken to represent the cost, then, as the gain is to be 20%, the selling price will be represented by 120.

As the selling price is to be 20% below the marked price, the selling price (120) will be .80 of the marked price.

Therefore, the marked price will be $\frac{100}{80}$ of $120 = 150$.

That is, the goods must be marked 50% above cost.

32. How many per cent above cost must a man mark his goods in order to take off 10%, and still make a profit of 17%?

33. How many per cent above cost must a man mark his goods in order to take off 12½%, and still make a profit of 12½%?

34. How many per cent above cost must a man mark his goods in order to take off 15%, and still make a profit of 15%?

35. How many per cent above cost must a man mark his goods in order to take off 33⅓%, and still make a profit of 33⅓%?

36. If 5% of the population of a town has been the increase in the preceding ten years, what per cent of the population ten years ago has been added?

37. If, in a population of 27,000,000, 13% are foreign-born, how many foreign-born are there? What is the ratio of the foreign-born to the native?

38. A man bought a horse for $70, and sold him for $80. What per cent did he gain? What per cent of the money received for the horse was gained?

39. If, by selling goods for 12½ per cent profit, a merchant clears $800, what was the cost of the goods, and for how much were they sold?

40. A man selling eggs at 40 cents a dozen clears 33⅓% on the cost; what was the cost? Another, selling at the same price, clears 33⅓% of his receipts; what did his eggs cost?

41. By selling a carriage for $117, a carriage-maker lost 10% of the cost. What ought he to have sold it for to make 10%?

42. A man gained in January 3% in weight, and in February lost 3%. What per cent of his weight on the first day of January is his weight on the first day of March?

43. 7 lbs. of a certain article lose 3 oz. in weight by drying. What per cent of the original weight is water?

44. 7 lbs. of a dry article have lost 3 oz. by drying. What per cent of the original weight was water?

45. A dry article was exposed to damp air, and absorbed 3 oz. of water; it then weighed 7 lbs. What per cent of its present weight is water?

46. If rosin is melted with 20% of its weight of tallow, what per cent of tallow does the mixture contain?

47. If 20% of a mixture of tallow and rosin is tallow, what per cent of the weight of the rosin is the weight of the tallow?

48. How many pounds of tallow must be mixed with 8½ lbs. of rosin in order that the mixture may contain 15% of tallow?

49. Nitrogen gas, under standard pressure and temperature, is $\frac{1}{8}$ of 1% of the weight of an equal volume of water. What is its specific gravity?

50. Oxygen gas is $\frac{1}{7}$ of 1% of the weight of an equal volume of water; what is its specific gravity? How many gallons of oxygen will it take to weigh as much as a pint of water? How many of nitrogen?

51. If common air consist of 4 volumes of oxygen to 13 of nitrogen, what is its specific gravity?

52. How many gallons of air weigh as much as a pint of water?

53. If, by heating iron 185° F., it expands $\frac{1}{8}$ of 1%, what will be the expansion of iron in passing from −20° F. to +120° F.?

54. A tubular iron bridge is 450 ft. long, and one end is fast to a pier. How much play must be allowed at the other end, if the iron expands at the above rate, and if the climate varies from −30° F. in winter to +130° F. in a July sun?

55. How much longer is 100 miles of iron rail at 118° F. than at 20° below zero?

348. In many kinds of commercial transactions the payment made is reckoned at a rate per cent.

COMMISSION AND BROKERAGE.

349. The *commission* paid to an agent for his services is generally reckoned at a rate per cent.

The sum left after the payment of the commission and other expenses is called *net proceeds.*

Commission paid to a *broker* is called *brokerage.*

(1) A commission merchant has consigned to him 50,000 lbs. of wool, which he sells at 50 cents a pound, and charges 2½% commission. He pays $125 for freight and $50 for cartage. Find the commission and net proceeds.

$.50 × 50,000	=		$25,000	
$25,000 × .025	=	$625		= commission.
Freight	=	125		
Cartage	=	50		
Total expenses	=		800	
			$24,200	= net proceeds.

(2) An agent receives $5000 with which to purchase goods, after deducting a commission of 1%. What is the amount of the commission, and how much remains for purchasing goods?

If 100 be taken to represent the amount to be paid for the goods, 1 will represent the commission, and 101 will represent the $5000. Therefore, the amount expended for goods will be $\frac{100}{101}$ of $5000 = $4950.50.

And 1% of $4950.50 = $49.50 = commission.

In selling, the commission is reckoned on *the amount received;* in buying, the commission is reckoned on *the amount paid.*

EXERCISE LVIII.

1. Find the brokerage, at ⅛ of 1%, to be paid on $10,450.
2. Find the commission on $2595, at 2½%.
3. An agent sells 200 bbls. of flour, at $6.25; 600 gals. molasses, at 65 cents; and charges a commission of 1¾%. What are the net proceeds?
4. A commission merchant received $1640 with which to buy corn, after deducting a commission of 2½%. What is the amount of his commission, and how many bushels of corn at 62½ cents a bushel can he buy?
5. A commission merchant sells a consignment of cotton for $5216. He pays $51 for freight and storage, and charges a commission of 2¼%. What are the net proceeds?
6. A consignment of butter was sold for $1570, of which $1546.45 were the net proceeds. What was the rate per cent of commission?
7. What are the net proceeds from the sale of 2250 bbls. of flour, at $6.25 a barrel, if the charges for freight and storage be 50 cents a barrel, commission for selling 2%, for guaranteeing payment 1½%?
8. A commission merchant sells 350 crates of peaches, at $2.60. If the commission be 4½%, find the net proceeds.
9. A man sells 420 acres of land, at $40 an acre, and charges 1¼% commission. What is his commission?
10. An agent, charging 4½% commission, receives for his services $313. Find the amount of his sales.
11. A merchant buys, through an agent, 730 yds. of carpeting, at $1.25 a yard, and pays the agent ¾ of 1% commission; the freight amounted to $7.37. At what price per yard must the carpeting be sold to realize a profit of 20%?

12. An agent sells a consignment of goods for $2100. He pays $33.50 for freight, and, reserving his commission, remits $2024.77. Find the rate of his commission.

13. A commission merchant has consigned to him 5000 lbs. of cotton, which he sells at 14 cents a pound, and charges 2% commission. With the net proceeds he buys cotton cloth, at 10 cents a yard, charging 1½% commission for buying. How many yards of cloth does he buy?

14. A commission merchant has consigned to him 500 bbls. of flour, which he sells at $5.50 a barrel, and charges 2½% commission; the expenses for freight, etc., amounted to $250. With the net proceeds he buys sugar, at 6¼ cents a pound, charging 2½% commission for buying. How much sugar does he buy, and what is the amount of his commissions?

15. A collector's commission for collecting taxes, at 1½%, is $206.55. What was the sum collected?

16. An agent received $2961 with which to purchase goods after deducting his commission at 5%. How much was his commission?

17. An agent buys 3100 bbls. of flour, at $4.50 a barrel, and charges 1½% commission. What is the amount of the bill including the commission?

18. A broker receives $6150 to invest in cotton, at 10¼ cents a pound. His commission is 2½%. How many pounds of cotton can he buy?

19. An agent sells 1100 bbls. of flour, at $4.50 a barrel, and charges 2½% commission. He invests the proceeds in steel, at 1½ cents a pound, charging 1½% commission. What is his entire commission, and how many tons of steel (2240 lbs. to a ton) does he buy?

INSURANCE.

350. In insurance a payment, called a *premium of insurance*, is made for a guarantee of a specified sum of money in the event of loss from fire or accident; and is reckoned at a rate per cent on the amount insured.

In life insurance an annual payment is made in order to secure a specified sum of money in the event of death.

(1) Find the premium for the insurance of a cargo worth \$36,000, at 6¼%.

$$\begin{array}{r} \$36000 \\ .06\frac{1}{4} \\ \hline \$2250.00 \end{array}$$

(2) For what sum should a cargo worth \$36,000 be insured at 6¼%, so that, in the event of loss, the owner may receive both the value of the cargo and the premium?

If 100 be taken to represent the sum to be insured, then the 6¼ will represent the premium; and 100 − 6¼, that is, 93¾, will represent the value of the cargo.

Hence, the sum to be insured will be

$$\$36,000 \div .93\frac{3}{4} = \$38,400.$$

EXERCISE LIX.

1. Find the premium of fire insurance for \$2650, at ½ of 1%.
2. Find the premium to be paid for insuring a person's life for \$2500, at an age for which the rate is 2¼%.
3. At 2¾%, what premium of insurance will be paid on a vessel worth \$36,400?
4. A vessel is worth \$12,052. Determine the sum to be insured, and the premium to be paid at 1¾%, so that in the event of loss the owner may receive both the value of the vessel and the premium.

5. The premium for insurance at 1¼% is $150. What is the amount insured?
6. If a premium of insurance at 2¾% amount to $28.60, what is the sum insured?
7. A vessel is so insured that if lost the owner may receive both the value of the vessel and the premium. The value of the vessel is $96,084, and the rate of insurance 1⅞%. Find the premium.
8. A building worth $8000 is insured at ⅝ of its value, at ⅛ of 1% per annum. What is the annual premium?
9. Four companies join in insuring a ship and cargo for $60,000. One company takes ⅓, at ⅗ of 1%; a second takes $10,000, at ¾ of 1%; a third, $15,000, at ⅝ of 1%; a fourth, the remainder, at ½ of 1%. How much is paid for insurance?
10. If the ship in the last problem receive damage to the amount of $4500, what ought each company to pay?
11. A man insures his life for $10,000, paying $350 a year in advance. He dies the day before the fifth premium was due. The company pay his widow $10,000. How much have they lost by him, if the interest gained on the premiums paid amount to $175?
12. A merchant shipped a cargo to London; and to cover both the cargo and the premium, he took out a policy of $100,800, at 3½%. What was the value of the cargo?
13. Three companies insure, at ¾ of its value, a building worth $16,000. The first company takes ⅓ the risk, at ¾ of 1%; the second, ⅖ of it, at ⅞ of 1%; and the third, the remainder, at ¾ of 1%. Find the total premium.
14. S. Williams pays $18.40 premium for insuring his house for ⅔ of its value at 1½%. What is the value of his house?

TAXES AND DUTIES.

351. *Taxes* on property are reckoned at a rate per cent on the assessed value of the property; and *duties* on imported goods are sometimes reckoned at a rate per cent on the cost in the country from which they are imported.

Ex. The assessed valuation of a town is $2,326,112; the number of polls is 800 (that is, the number of persons who pay a poll-tax); the amount required for expenses is $15,830.56. If each poll pays a tax of $1.50, and the sum to be received from the State is estimated at $3000, what is the rate per cent of tax on the property? What is the tax of S. Jones, who pays for one poll, and has property valued at $8216?

The amount of poll-taxes = 800 × $1.50 = $1200.
$15,830.56 – $4200 = $11,630.56 to be assessed on property.
$11,630.56 ÷ $2,326,112 = .005 = ½ of 1%.
That is, the tax is 5 mills on a dollar, or $5 on $1000.
Therefore, S. Jones's property-tax is .005 of $8216 = $41.08.
Total tax = $41.08 + $1.50 = $42.58.

EXERCISE LX.

1. If James Brown be assessed $2500 on his house and $5200 on personal property, and pays for 2 polls at $1.50 each, how much will his tax be, the rate being $12.18 on $1000?
2. If the rate of tax be $12.25 on $1000, and the tax be $11,788.50, what is the valuation?
3. If the assessed valuation of a town be $1,777,000, and the property-tax be $6870, what is the rate on $1000.
4. What sum must be assessed, in order that $15,000 shall remain after paying a commission of 2% for collecting the taxes?

5. A tax of $1857.60 is levied upon a school district for building a school-house. The assessed valuation of the district is $1,935,000. What is the tax on property valued at $6250?

6. In a certain town there are 1350 polls. The assessed value of the real estate is $713,250; of the personal property is $738,954; the poll-tax is $2. The tax on property is 1⅛%. But only 96% of the tax can be collected, and the collector is paid 2½% of the amount collected. How much does the town receive from the taxes?

7. What is the duty, at 20% *ad valorem* (that is, 20% of the cost), on 320 boxes of raisins, each containing 40 lbs., and costing 8 cents a pound?

8. What is the duty, at 6 cents a gallon, on 420 hhds. of molasses, 63 gals. in a hogshead?

9. At 40%, what is the duty on 300 tons of iron (2240 lbs. to a ton) invoiced at 1½ cents a pound?

10. Paid $1360.80 duty on 300 hhds. of molasses, each containing 63 gals., at 25 cents a gallon. What was the rate per cent of duty?

11. A sugar refiner imports 50 hhds. of sugar weighing 480 lbs. each, and 120 hhds. of molasses containing 63 gals. each. What is the amount of the duties, if the sugar pay 3 cents a pound, and the molasses 8 cents a gallon, an allowance being made on the sugar of 10%, and 2% on the molasses?

12. An importer paid $825 duty on an invoice of silks, the duty being 24%. But damages of 15% were allowed at the custom-house. What was the entire cost of the goods?

13. Paid $325 duty on goods which had been damaged; allowance for damage is 24%, and the duty was 24%. What was the invoice price of the goods?

CHAPTER XVI.

INTEREST AND DISCOUNT.

352. **Interest** is the payment made for the use of money.

The interest to be paid for a given sum differs from the payments considered in the last chapter; in that it *depends* on the *time* for which the sum is loaned as well as on the *rate per cent* charged.

353. The sum loaned is called the **principal.** The principal and interest, added together, are called the **amount.**

SIMPLE INTEREST.

354. If 100 be taken as the representative of the principal, the rate will represent *the interest for one year;* the product of *the rate per cent by the number of years* will represent the *whole interest,* and this added to 100 will represent the amount.

Thus, if the time be 3 years and the rate per cent 6, the interest will be represented by 18, and the amount by 118.

(1) Find the interest on \$1024, for 2 yrs. 8 mos., at $5\frac{1}{2}\%$.

Time, 2 yrs. 8 mos. = $2\frac{2}{3}$ yrs.

$$\begin{array}{r l}
\$1024 & \\
.05\frac{1}{2} & \\
\hline
512 & \\
5120 & \\
\hline
\$56.32 & = \text{interest for 1 yr.} \\
2\frac{2}{3} & = \text{2 yrs. 8 mos.} \\
\hline
3755 & \\
11264 & \\
\hline
\$150.19 & \textit{Ans.}
\end{array}$$

(2) Find the interest of $1020, for 5 yrs. 11 mos. 18 dys., at 6%.

The interest at 6% for 1 year is .06 of the principal.
The interest for 1 month is $\frac{1}{12}$ of .06 = .005 of the principal.
The interest for 1 day is $\frac{1}{30}$ of .005 = $\frac{1}{6}$ of .001 of the principal.
Hence, the interest for

5 yrs. = 5 × .06 = .30
11 mos. = 11 × .005 = .055
18 dys. = 18 × $\frac{1}{6}$ of .001 = .003
5 yrs. 11 mos. 18 dys. = .358 of the principal.
.358 of $1020 = $365.16. *Ans.*

The method employed in the last example may be employed for any rate per cent, by first finding the interest at 6%, and then taking such a part of the interest as the given rate is of 6%. Thus, the interest at 4½% = $\frac{4\frac{1}{2}}{6}$ = $\frac{3}{4}$ of the interest at 6% = interest at 6% − $\frac{1}{4}$ of itself. The interest at 8% is $\frac{8}{6}$ = $\frac{4}{3}$ of the interest at 6% = interest at 6% + $\frac{1}{3}$ of itself.

In most business transactions the time for which interest is required is 1, 2, 3, or 4 months (30 dys. being reckoned 1 mo.), and the rate of interest is ½% a month. Hence, the interest on a given sum for 2 mos. (or 60 dys.) is found by moving the decimal point two places to the left; for 1 mo., 3 mos., 4 mos., by moving the decimal point two places to the left, and multiplying by ½, 1½, and 2, respectively. Thus, the interest on $1250 for 2 mos. is $12.50; for 1 mo., $6.25; for 3 mos., $18.75; for 4 mos., $25.

EXERCISE LXI.

Find the interest of:

1. $680.40 for 2 yrs. 4 mos. 6 dys., at 6%.
2. $25.625 for 30 dys., at 6%.
3. $85.85 for 1 yr. 7 mos. 21 dys., at 6%.

4. $1100 for 3 yrs. 4 mos., at 5%.
5. $1275 for 3 yrs. 2 mos. 15 dys., at 8%.
6. $475.16 for 27 dys., at 4½%.
7. $1290.50 for 60 dys., at 6%.
8. $125 for 1 yr. 2 mos. 2 dys., at 9%.
9. $250.80 for 10 mos. 10 dys., at 3½%.
10. $258.85 from Mar. 6, to June 24, at 5%.
11. $380 for 2 yrs. 11 mos. 27 dys., at 4½%.
12. \$475.05 for 1 yr. 9 mos. 14 dys., at $7\frac{3}{10}\%$.
13. $725.40 for 11 mos. 24 dys., at 5¼%.
14. $680.50 for 2 yrs. 6 dys., at 5%.
15. $630.50 for 90 dys., at 6%.
16. $547.60 from Feb. 20 to Dec. 5, at 6½%.
17. $875 from May 5, 1880, to June 21, 1881, at 5½%.
18. $758.50 from Jan. 5 to July 1, at 4½%.
19. $342.42 from Feb. 5, 1879, to Mar. 15, 1881, at 7%.
20. $540 from Mar. 5 to Sept. 21, at 3½%.

Find the amount of:

21. $431.50 for 2 yrs. 8 mos., at 4½%.
22. $476.50 from July 5, 1880, to Feb. 9, 1881, at 4%.
23. $319.20 from April 7 to Aug. 31, at 3¼%.
24. $6460 from June 15, 1878, to May 7, 1880, at 4¼%.
25. $150 from Aug. 5, 1879, to Mar. 17, 1881, at 7%.
26. $527.20 from Jan. 1 to Nov. 20, at 4½%.
27. $1250 from Nov. 15, 1880, to Mar. 1, 1881, at 5%.
28. \$624.36 from Mar. 5 to Dec. 20, at $7\frac{3}{10}\%$.
29. $12,260 from May 6 to Oct. 24, at 3¾%.
30. $11,216 from Oct. 20 to Dec. 31, at 1% a month.

NOTE. In business a year is reckoned at 360 days in computing interest *for a time less than a year expressed in months and days;* hence, the interest is $\frac{5}{365}$ or $\frac{1}{73}$ too great. But general governments take the number of days between the two given dates, and reckon for the interest such a part of a year's interest as this number of days is of 365 days.

355. It is often required to find the rate, time, or principal, when two of these and the interest (or amount) are given.

(a) *When the principal, interest (or amount), and time are given, to find the rate per cent.*

(1) At what rate per cent will $480 produce $72 in three years?

Interest on $480 for 3 years is $72;
on $480 for 1 year is $\frac{1}{3}$ of $72;
on $1 for 1 year is $\frac{1}{480}$ of $\frac{1}{3}$ of $72 = $.05.
Therefore, rate required is 5%.

(2) At what rate per cent will $8432 amount to $9437.516 in 2 yrs. 7 mos. 24 dys.?

The interest is $9437.516 − $8432 = $1005.516.
The time is 2 yrs. 7 mos. 24 dys. = 2.65 yrs.
Interest on $8432 for 2.65 yrs. is $1005.516;

$$\text{on } \$8432 \text{ for 1 yr. is } \frac{\$1005.516}{2.65};$$

$$\text{on } \$1 \text{ for 1 yr. is } \frac{\$1005.516}{2.65 \times 8432} = \$.04\tfrac{1}{2}.$$

Hence, rate required is $4\frac{1}{2}$%.

The rate may also be found by forming a proportion with the representative numbers. Thus,

$$\begin{array}{cccc} \text{Capital.} & \text{Capital.} & \text{Interest.} & \text{Interest.} \\ 8432 : & 100 :: & 1005.516 : & \text{what?} \end{array}$$

The fourth term will be 11.925.

Hence, the interest, which = rate × time, is represented by 11.925.

$$\text{Therefore, the rate} = \frac{11.925}{2.65} = 4.5.$$

That is, the rate is $4\frac{1}{2}$%.

Find the rate per cent:

31. When the interest on $326 for 15 yrs. is $220.05.
32. When the interest on $372.50 for 18 yrs. is $301.725.
33. When $245 amount to $252.96¼ for 9 mos.
34. When the interest on $235.25 is $70.575 for 5 yrs.
35. When $363.125 amount to $371.598 for 7 mos.
36. When the interest on $249.43¾ is $49.88¾ for 5 yrs. 4 mos.
37. When $350 amount to $406.70 for 3 yrs. 7 mos. 6 dys.
38. When the interest on $6875 is $72.05 for 90 dys.
39. When the interest on $642 is $10.70 for 5 mos.
40. When the interest on $8432 for 2 yrs. 7 mos. 23 dys. is $1339.28.
41. When a sum of money is doubled in 14 yrs.
42. When an investment for 5 yrs. 2 mos. produces a sum equal to ⅖ of the capital.
43. When an investment for 3 yrs. 1 mo. 15 dys. produces a sum equal to ⅛ of the capital.

(b) *When the principal, interest (or amount), and rate per cent are given, to find the time.*

Ex. In what time will the interest on $8432 amount to $1005.516, at 4½%?

Interest on $8432, at 4½%, for 1 yr. is .045 of $8432 = $379.44.

Therefore, the number of years will be $\frac{\$1005.516}{\$379.44} = 2.65$.

And 2.65 yrs. = 2 yrs. 7 mos. 24 dys. *Ans.*

Find the time in which:

44. The interest on $450 will amount to $72, at 4%.
45. The interest on $487.50 will amount to $39, at 4%.
46. The interest on $238.75 will amount to $64.46, at 4½%.
47. The sum of $793.875 will amount to $805.84, at 5½%.
48. A sum of money will double itself at 4%.

49. The sum of $10 will amount to $17, at 6%.
50. The sum of $502.67 will amount to $578.07, at 4½%.
51. The interest on $537.50 will amount to $80.625, at 4%.
52. The interest on $6875 will amount to $75.05, at 4¼%.
53. The interest on $8520 will amount to $1746.60, at 6%.

(c) *When the interest, time, and rate are given, to find the principal.*

Ex. What principal will in 2 yrs. 7 mos. 24 dys. produce $1005.516 interest, at 4½%?

2 yrs. 7 mos. 24 dys. = 2.65 yrs.

Interest for 1 yr. $= \frac{\$1005.516}{2.65} = \$379.44.$

Interest on $1 for 1 yr., at 4½% = $.045.

Hence, principal required $= \frac{379.44}{.045}$ of $1 = $8432.

Find the principal that:

54. Will produce $90 interest in 3 yrs., at 4%.
55. Will produce $63 interest in 3 yrs., at 6¼%.
56. Will produce $100 interest in 8 yrs. 6 mos., at 5%.
57. Will produce $1746.60 interest in 3 yrs. 5 mos., at 6%.
58. Will produce $12 interest in 7 mos., at 5%.
59. Will produce $50 interest in 228 dys., at 4½%.
60. Will produce $1339.28 interest in 2 yrs. 7 mos. 24 dys., at 6%.
61. Will produce $1312.65 interest in 2 yrs. 3 mos., at 6%.
62. Will produce $750 interest in 3 yrs. 8 mos., at 5%.

(d) *When the amount, time, and rate per cent are given, to find the principal.*

Ex. Find the principal that will amount to $748.125 in 3 yrs. 6 mos., at 4%.

3 yrs. 6 mos. = 3½ yrs.

If the principal be represented by 100, the interest will be represented by 3½ × 4 = 14, and the amount will be represented by 114. Hence, the principal = $\frac{100}{114}$ of $748.125 = $656.25.

Find the principal that will amount:

63. To $840 in 3 yrs., at 4%.
64. To $901.1384 in 2 yrs. 6 mos., at 4⅛%.
65. To $6000 in 21 dys., at 5%.
66. To $297.60 in 8 mos., at 6%.
67. To $6378.75 in 1 yr. 1 mo., at 5%.
68. To $21,047.95 in 1 yr. 7 mos. 21 dys., at 4½%.
69. To $185.09 in 2 yrs. 3 mos. 18 dys., at 5%.
70. To $659.40 in 2 yrs. 11 mos. 15 dys., at 6%.
71. To $9437.516 in 2 yrs. 7 mos. 24 dys., at 4½%.
72. To $10,266.60 in 3 yrs. 5 mos., at 6%.

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73. What is the interest of $195 for 2 yrs. 2 mos. 2 dys., at 6½%?
74. At what rate per cent will $1025.20 produce $25.72 in 4 mos. 9 dys.?
75. The principal is $653; the interest $5.52; the rate 8%. Find the time.
76. Find the amount of $520 for 2 mos. 3 dys., at 4½%.
77. What sum bearing interest at 4½% will yield an annual income of $1000?
78. How long will it take $4000 to produce $625 interest, at 5½%?
79. At what rate per cent will $3000 produce $250 interest in 1 yr. 2 mos. 24 dys.?
80. Find the interest of $1721.84 from April 1 to Nov. 12, at 4½%.
81. How long must $3904.92 be on interest to amount to $4568.76, at 5%?
82. Find the interest of $137.60 from July 3 to Dec. 12, at $7\frac{3}{10}$%.
83. Find the interest of $680.20, at 7½%, for 73 dys., reckoning 365 dys. for a year.

Discount.

356. When the holder of a promissory note sells the note to a bank, or other purchaser, the sum paid by the bank is called the **proceeds** or **avails** of the note, and the difference between the sum named in the note and the proceeds is called the **discount.**

357. Discount is reckoned at so much per cent, and the per cent is called the **rate of discount.**

358. Questions in discount are calculated like questions in simple interest, the terms used being *discount* instead of *interest*, and *rate of discount* instead of *rate of interest.*

Note. The sum named in the note should be written in words, and is called the *face* of the note. The person signing the note is called the *maker;* a person who writes his name on the back of the note is called an *endorser*, and is responsible for the payment of the note.

A note, to be legal, must contain the words "*value received*"*;* to be negotiable, must be made payable to the *bearer*, or *to the order* of some person who must endorse the note.

When a note bears interest, the discount is computed on the face of the note with the interest added.

A note is *nominally* due at the expiration of the time named in the note, but it does not *mature*, that is, become *legally* due, until three days after this time. These three days are called **days of grace.** And the discount is computed on the time between the day the note is discounted and the day of its maturity.

When the time is expressed in *days*, the day of maturity is found by counting forward from the date of the note the *number of days* named in the note, and the three days of grace. When the time is in *months*, the day of maturity is found by counting the *number of calendar months*, and the three days of grace. When a note falls due on Sunday, or a legal holiday, it is payable on the day previous.

A **protest** is a notice in writing by a notary public to the endorsers that a note has not been paid. If a note be not protested on the last day of grace the endorsers are released from their obligation.

Find the *day of maturity*, the *time to run* (from the day the note is discounted), the *discount*, and the *proceeds* of the following notes:

(1) $520.16. BOSTON, Jan. 12, 1881.

Sixty days after date I promise to pay to the order of G. L. Gage five hundred twenty and $\frac{16}{100}$ dollars, for value received.

Discounted at 6%, Feb. 1. JOHN KELLEY.

Counting 60 days, from Jan. 12, there are 19 in January, 28 in February, and 13 in March.

Therefore the note becomes due Mar. $^{13}/_{16}$. (13 denotes the time it is nominally due, 16 the time it is legally due.)

The time to run is 27 days in February and 16 in March = 43 days.

The discount is the interest on $520.16 for 43 days, at 6%. Therefore,

The discount is $.007\frac{1}{6}$ of $520.16 = $3.73.

The proceeds is $520.16 − $3.73 = $516.43.

(2) $650. CINCINNATI, Nov. 3, 1880.

Six months from date we jointly and severally promise to pay to the order of Charles Fall six hundred and fifty dollars, value received, with interest at six per cent.

Discounted at 7%, Jan. 3, 1881.

JOHN HENDERSON.
JAMES HENDRICKS.

Interest on note for 6 mos. 3 dys., = $19.82.
Amount of note $650 + $19.82 = $669.82.
Day of maturity, May $^{3}/_{6}$, 1881.
Time to run, 4 mos. 3 dys.
Discount on $669.825, at 7%, for 4 mos. 3 dys. = $16.02.
Proceeds is $669.82 − $16.02 = $653.80.

EXERCISE LXII.

Find *day of maturity*, the *time to run*, the *discount*, and *proceeds* of the following notes:

1. $750. NEW YORK, Jan. 1, 1881.

Four months from date I promise to pay to the order of James Fay seven hundred and fifty dollars, value received.

Discounted at 7%, Jan. 12. JOHN PRAY.

2. $4325.50. BOSTON, Jan. 3, 1881.

Sixty days from date I promise to pay to James Finn, or order, four thousand three hundred twenty-five and $\frac{50}{100}$ dollars, value received.

Discounted at 6½%, Jan. 6. GEORGE BELLOWS.

3. $1340.70. RICHMOND, Va., Jan. 6, 1881.

Ninety days from date I promise to pay to the order of Peter Bright thirteen hundred forty and $\frac{70}{100}$ dollars, value received.

Discounted at 7%, Jan. 26. GEORGE WRIGHT.

4. $1456.30. CHARLESTON, S.C., Jan. 19, 1881.

Three months after date I promise to pay to the order of John George fourteen hundred fifty-six and $\frac{30}{100}$ dollars, value received.

Discounted at 5%, Feb. 1. JOHN WALDORF.

5. $4550.36. DETROIT, Mich., Feb. 2, 1881.

Four months after date I promise to pay to the order of John Callender four thousand five hundred fifty and $\frac{36}{100}$ dollars, value received.

Discounted at 5½%, Feb. 16. JAMES BARTON.

6. $5000. CHICAGO, Ill., Oct. 4, 1880.

Six months after date I promise to pay to John Adams or order five thousand dollars, value received, with interest at seven per cent.

Discounted at 8%, Dec. 31. WILLIAM DUNN.

7. $4760. MILWAUKEE, Wis., Jan. 1, 1881.

Ninety days after date I promise to pay to the order of James Pike four thousand seven hundred and sixty dollars, value received.

Discounted at 7½%, Feb. 15. WILLIAM CLEMENT.

8. $2017.85. KANSAS CITY, Mo., Jan. 14, 1881.

Three months after date I promise to pay to the order of John Brown two thousand seventeen and $\frac{85}{100}$ dollars, value received.

Discounted at 10%, Mar. 1. TIMOTHY BRUCE.

9. $652.45. CONCORD, N H., Jan. 25, 1881.

Five months after date I promise to pay to the order of Charles Barrett six hundred fifty-two and $\frac{45}{100}$ dollars, value received, with interest at six per cent.

Discounted at 4½%, Mar. 15. WILLIAM KIMBALL.

10. $9040. BALTIMORE, Md., Jan. 19, 1881.

Sixty days from date I promise to pay to the order of Charles Carroll nine thousand and forty dollars, value received.

Discounted at 5½%, Feb. 16. JAMES MONROE.

11. $215. AUGUSTA, Me., Jan. 28, 1881.

Thirty days after date, I promise to pay to the order of James Fogg two hundred and fifteen dollars, value received.

Discounted at 6%, Feb. 3, JOHN MOSES.

359. If it be required to determine the face of a note that will yield a given sum when discounted at a bank, the method will be as follows:

Ex. For how much must a 3-months note, without interest, be made, that it may yield $500 when discounted at 6%?

The discount on $1 for 3 mos. 3 dys. is $.0155.
Proceeds on $1 is $1 − $.0155 = $.9845.
Therefore, the face required for $500 is $500 ÷ .9845,
= $507.87. *Ans.*

12. Find the face of a note at 90 dys. that will realize $850 when discounted at 7%.

13. Find the face of a note at 4 mos. that will realize $1600 when discounted at 5½%.

14. Find the face of a note at 30 dys. that will realize $1200 when discounted at 6½%.

15. Find the face of a note at 60 dys. that will realize $4000 when discounted at 8%.

16. Find the face of a note at 2 mos. that will realize $4500 when discounted at $7\frac{3}{10}$%.

17. Find the face of a note at 3 mos. that will realize $1100 when discounted at 7%.

PRESENT WORTH AND DISCOUNT.

360. The *present worth* of a sum of money due at the end of a given time, is the sum that, put at interest for the given time, will amount to the given sum.

Thus, $50 will, in 2 yrs., at 6%, amount to $56. And $56 to be paid at the end of 2 yrs. is equal in value to $50 paid now. Hence, $50 is regarded as the present worth of $56 to be paid in 2 yrs.

361. The difference between the given sum and its present worth is called the true discount.

362. It is evident that this discount of the given sum is the interest of its present worth.

The operation of finding the present worth of a sum of money at a given rate of interest is the same as the operation of finding the principal when the amount, time, and rate per cent are given. § 355.

Ex. Find the present worth and discount of $5000 due at the end of 3 yrs. 6 mos., at 6%.

If 100 be taken to represent the present worth, the discount will be represented by $3\frac{1}{2} \times 6 = 21$.

The given sum will be represented by $21 + 100 = 121$.

Hence, the present worth is $\frac{100}{121}$ of $5000 = $4132.23; and the discount is $5000 − $4132.23 = $867.77.

18. Find the present worth of $500 due in 11 mos., at 5%.
19. Find the present worth and discount of $3334.62 due in 2 yrs., at $4\frac{1}{2}$%.
20. Find the present worth and discount of $4261.33 due at the end of 1 yr. 6 mos. at 6%.
21. Find the present worth and discount of $2416.50 due in 7 mos., at 5%.
22. Find the present worth of $678.40 due in 16 mos., at $4\frac{1}{2}$%.
23. Find the present worth and discount of $574.17 due in 2 yrs. 3 mos., at $5\frac{1}{3}$%.
24. Find the present worth and discount of $625.13 due in 8 mos., at $7\frac{3}{10}$%.
25. Find the present worth and discount of $715.20 due in 1 yr. 4 mos., at $3\frac{1}{2}$%.

NOTE. This discount is called *true* discount to distinguish it from discount used in business transactions. Discount in business always means either so many per cent off without reference to time, as when a merchant is paid before he proposes to demand payment; or so many per cent off reckoned at the current rate of interest for a given time, as when a note is discounted at a bank.

PARTIAL PAYMENTS.

363. When settlements of accounts between two merchants are made at the expiration of a *year, or less*, it is customary to reckon interest on the several items in the accounts to the time of settlement. Also, when partial payments are made, and endorsed on a note that contains the words "**with interest,**" and that is paid in full within a year, it is usual to compute the interest on the principal and on each of the payments to the time of settlement.

(1) Andrew Hall buys of John Paul $400 worth of goods, at 30 dys. At the end of 3 mos. he pays $200, and the balance 2 mos. later. Find the balance.

The time between the end of 30 dys. and the time of settlement is 4 mos. Therefore, interest is reckoned on the $400 for 4 mos. and on the $200 for 2 mos.

$400 + 4 mos. interest = $408
$200 + 2 mos. interest = 202
Therefore, balance due is $206

(2) A man holds a note for $500, dated Jan. 1, 1880, on which are endorsed partial payments as follows: Mar. 1, 1880, $50; Oct. 1, 1880, $25; Nov. 1, 1880, $400. What is due Jan. 1, 1881, interest at 7%?

Amount of $500 for 1 yr. at 7% is $535.00
Amount of $50 for 10 mos. at 7% = $52.92
Amount of $25 for 3 mos. at 7% = 25.44
Amount of $400 for 2 mos. at 7% = 404.67
483.03
Balance due, $51.97

364. This method is in accordance with what is called the **Merchants' Rule.**

26. If I buy goods Jan. 10, at 30 dys., for $218; Feb. 5, at 60 dys., for $421; and pay Feb. 10, $200, Mar. 17, $50; what is due Apr. 25, interest at 6%?

27. A note for $618.75, dated Apr. 17, 1880, payable on demand, bears the following endorsements: June 5, $126.50; Aug. 20, $137.25; Nov. 17, $210. What is due Jan. 1, 1881, reckoning interest at 6%?

28. A note for $1000, dated Apr. 1, 1880, payable on demand, with interest at 7%, bears the following endorsements: May 6, $200; July 5, $225.37; Oct. 18, $322. What is due Jan. 1, 1881?

29. A note for $835.25, dated July 1, 1880, payable on demand, with interest at 6½%, bears the following endorsements: Aug. 20, $157.50; Sept. 21, $180.25; Oct. 5, $200; Dec. 1, $80. What is due Jan. 1, 1881?

When a note runs *longer than a year*, and has endorsements, the Merchants' Rule works injustice to the holder of the note, as may be seen by an extreme case: Thus, if the interest due at the end of each year be endorsed on a note of $1000, at 6% interest, the amount of the interest payments in about 24 years will cancel the whole note, without the payment of one cent of the principal. Hence,

365. When a note that contains the words "**with interest**" runs longer than a year, and partial payments have been made, the interest is computed by a rule adopted by the Supreme Court of the United States, and therefore called

THE UNITED STATES RULE.

Find the amount of the principal to the time when the payment, or sum of the payments, equals or exceeds the interest.

From this amount deduct the payment or sum of the payments.

Consider the remainder as a new principal, and proceed as before.

Ex. A note of $1520, dated May 20, 1880, and drawing interest at 6%, had payments endorsed upon it as follows: Oct. 2, 1880, $300; Feb. 26, 1881, $25; Apr. 2, 1881, $570; Aug. 8, 1881, $600. Find the amount due Dec. 6, 1881.

yr.	mos.	dys				
1880	10	2			$1520	
1880	5	20			.022	
	4	12	.022		$33.44	1st interest.
					1520.00	
$300.					$1553.44	
					300.00	1st payment.
1881	2	26			$1253.44	2d principal.
1880	10	2			.024	
	4	24	.024	$25	$30.08	2d interest.
					$1253.44	2d principal.
$25.					.006	
				$570	$7.52	3d interest.
1881	4	2			30.08	2d interest.
1881	2	26			1253.44	
	1	6	.006		$1291.04	
					595.00	2d & 3d payments.
$570.					$696.04	3d principal.
					.021	
1881	8	8			$14.62	4th interest.
1881	4	2			696.04	
	4	6	.021		$710.66	
					600.00	
$600.					$110.66	4th principal.
					$.019\frac{2}{3}$	
1881	12	6			$2.18	5th interest.
1881	8	8			110.66	
	3	28	$.019\frac{2}{3}$		$112.84	*Answer.*

In the first place, find the difference in time between each pair of consecutive dates. At the right of the result in each case put the corresponding decimal multiplier for the interest at 6%, and put the corresponding payment below.

Generally it can be determined *mentally* whether one or more payments must be taken to make a sum equal to or greater than the interest. If two or more payments are required, the corresponding decimal multipliers may be added and the result taken for the multiplier. Thus, it is evident that .024 of $1253.44 is more than $25; therefore, .024 + .006 = .03, may be taken for the multiplier, which will give for the interest $37.60. To this the principal is added, and from the amount the sum of the payments is subtracted.

When the rate is greater or less than 6% the several *interests* must be increased or diminished according to the given rate.

30. A note of $2000, dated Jan. 22, 1880, and drawing interest at 6%, had payments endorsed upon it as follows: May 20, 1880, $100; July 20, 1880, $325; Nov. 2, 1880, $20; Dec. 23, 1880, $125. Find the balance due Mar. 1, 1881.

31. A note of $1662.50, dated Jan. 15, 1880, and drawing interest at 6½%, had payments endorsed upon it as follows: Apr. 30, 1880, $25; June 24, 1880, $25; Sept. 2, 1880, $625; Jan. 31, 1881, $700. Find the balance due May 12, 1881.

32. A note of $4560, dated Jan. 22, 1879, and drawing interest at 7%, had payments endorsed upon it as follows: Jan. 10, 1880, $2000; Aug. 31, 1880, $500; Jan. 15, 1881, $1200; Mar. 4, 1881, $860. Find the balance due June 15, 1881.

33. A note of $785.50, dated Jan. 30, 1879, and drawing interest at 5%, had payments endorsed upon it as follows: July 17, 1879, $100; Jan. 29, 1880, $100; Dec. 31, 1880, $20; Mar. 16, 1881, $300; June 14, 1881, $50. Find the balance due July 23, 1881.

34. A note of $300.25, dated Aug. 4, 1879, and drawing interest at 6½%, had payments endorsed upon it as follows: Oct. 14, 1879, $100; July 21, 1880, $100; Oct. 11, 1880, $50; Jan. 18, 1881, $50. Find the amount due July 22, 1881.

COMPOUND INTEREST.

366. When a note contains the words "**with interest annually,**" and the interest is not paid at the time it is due, the interest is usually added to the principal; and new principals are thus formed at regular intervals of time.

367. The interest may be compounded with the principal (that is, made a part of the principal), annually, semi-annually, quarterly, etc., according to agreement.

Ex. Find the compound interest of $800 for 2 yrs. 3 mos. 15 dys., at 7%.

	$800	
	.07	
	$56	1st interest.
	800	
	856	2d principal.
	.07	
	$59.92	2d interest.
	856.00	
	$915.92	3d principal.
3 mos. 15 dys.	.0175	
	6)16.03	
	2.67	
	$18.70	3d interest.
	915.92	
	$934.62	amount.
	800.00	
	$134.62	interest.

Hence, the compound interest, at 7%, for the given time is $134.62.

368. If the given time be not an integral number of years, the amount is found for the number of entire years, and then the amount of this for the fractional part of a year.

35. Find the amount of $356.25 in 4 yrs., at 5% compound interest.

36. Find the amount of $637.50 in 2 yrs. 6 mos., at 4% compound interest.

37. Find the compound interest of $800 in 3 yrs. 9 mos., at 6%.

38. Find the compound interest of $39.35 in 1 yr. 9 mos., at 5%.

If the interest be payable semi-annually, quarterly, etc., the half, quarter, etc., of the rate per cent must be used, and the amount obtained for each half-year, quarter-year, etc.

39. Find the compound interest of $300 in 2 yrs., at 4%, interest being payable semi-annually.

40. Find the compound interest of $525 in 1 yr. 6 mos., at 5%, interest being payable quarterly.

41. Find the compound interest of $10,000 in 6 mos., at 6%, interest being paid monthly.

Ex. What principal will in 3 yrs. produce $780.40 compound interest, at 4%?

The amount of $1 for 1 yr. at 4% is $\$1 \times 1.04$;
for 2 yrs. is $\$1 \times 1.04 \times 1.04 = \$1 \times 1.04^2$;
for 3 yrs. is $\$1 \times 1.04^2 \times 1.04 = \$1 \times 1.04^3$.
Hence, the amount of $1 for 3 yrs. at 4% is $1.124864.
The interest is $\$1.124864 - \$1 = \$.124864$.
$\$780.40 \div .124864 = \$6250$.

42. What principal will amount to $137.81 in 2 yrs., at 5% compound interest?

43. What principal will amount to $1860.96 in 3 yrs., at 6% compound interest?

44. What principal will amount to $1500 in 1 yr., at 4% compound interest, payable quarterly?

45. What principal will produce $100 in 1 yr. 6 mos., at 6% compound interest, payable semi-annually?

CHAPTER XVII.

STOCKS.

369. The name **stock** is applied to the capital of banks, railroads, and other incorporated companies.

370. The capital of a company is usually divided into **shares**, of which the *original value* is $100, or some other fixed sum; but the *market value* at any time is the current price per share.

371. When the market value of stock is equal to its original value, it is said to be **at par.** In quotations of stocks par is generally represented by 100; and when stock is quoted at above 100, it is said to be at a premium; below 100, at a discount. The premium or discount is the difference between the quotation and 100.

Thus, when the price of a stock on a given day is 91, or, as it is commonly expressed, when the stock is *at* 91, the meaning is, that $100 stock costs on that day $91 money. Or that if 100 be the representative of any quantity of stock, 91 will represent the corresponding value in money. In this case the stock is said to be 9% discount.

The buying and selling of stocks is conducted through the agency of stock-brokers, who receive a brokerage on the stock. The brokerage is generally reckoned at $\frac{1}{8}$ of 1% on the *par value* of the stock. Thus, if a broker sells stock for a person at 91, that person receives $90\frac{7}{8}$; and if he buys stock for a person at 91, that person pays $91\frac{1}{8}$.

(1) How much would be received for 52 shares of stock, $100 each, at $89\frac{1}{2}$?

$\frac{1}{8}$ will represent the brokerage.

$89\frac{1}{2} - \frac{1}{8} = 89\frac{3}{8}$, price to the seller.

Hence, 1 share will bring $\$89\frac{3}{8}$; and 52 shares, $52 \times \$89\frac{3}{8}$

= $4647.50. Ans.

(2) What amount of stock at 84½ may be bought for $9393.375?

If ⅛ be the brokerage,
84½ + ⅛ = 84⅝, price to the buyer.
Hence, $.84⅝ buys $1 stock, and $9393.375 will buy
$9393.375 ÷ .84⅝ = $11,100. *Ans.*

(3) What is the quoted price of stock when $42,464.25 is paid for $46,600 stock?

If $46,600 stock cost $42,464.25,
then $1 stock costs $42,464.25 ÷ 46,600 = $.91⅛.
Hence, the stock is quoted at 91⅛ − ⅛ brokerage = 91. *Ans.*

EXERCISE LXIII.

In the following examples the prices of stocks include brokerage unless otherwise stated:

1. Find the cost of $4000 stock, at 109⅞.
2. Find the cost of $2500 stock, at 98.
3. Find the cost of $3900 stock, at 78⅛.
4. Find the cost of $4700 stock, at 100½.
5. Find the cost of $1250 stock, at 87⅝, brokerage ⅛.
6. How much bank stock, at 75¼, may be bought for $8729?
7. How much railroad stock, at 91⅛, may be bought for $\$4237\frac{5}{16}$?
8. How much railroad stock may be bought for $6305, at 121¼?
9. How much railroad stock may be bought for $5137.50, at 102¾?
10. How many $100 railroad shares, at 68¾, may be bought for $1650?
11. What must be the price of stock, in order that $9200 stock may be bought for $8970?

12. If $1500 stock be bought for $1374.375, what is the price of the stock?

(1) What income will be derived from $18,700 6% stock?

.06 of $18,700 = $1122. *Ans.*

(2) How much 5% stock must be bought to give an income of $360?

Since $.05 is derived from $1 stock, $360 will be derived from $360 ÷ .05 = $7200. *Ans.*

13. What income will be derived from $29,700 4% stock?

14. Find the income from $4500 6% stock.

15. How much will a person receive from $9400 railroad stock if a dividend of 3½% be declared?

16. How much 8% stock must be bought to give an income of $2400?

17. A person receives $343 as his semi-annual dividend from a 7% stock. How much stock does he hold?

18. Find the total income of a person whose property consists of $3000 6% stocks and $8200 7% stocks.

19. Find the rate of dividend paid by some stock, when a holder of $24,600 receives $924.50.

20. Find the rate per cent at which $11,100 will yield a semi-annual return of $499.50.

Ex. If $10,250 be invested in 6% stock, at 102½, what income will be obtained?

$102½ is the cost of $100 stock.
Hence, $10,250 = cost of $10,250 ÷ 1.025 = $10,000 stock.
And .06 of $10,000 = $600. *Ans.*

21. If $19,500 be invested in 4% stock, at 91, what income will be received?

22. Find the income on $7000 when invested in 4% stocks, at 103¼.

23. What income will be derived from $6800 if it be invested in 7% stocks, at 130?

24. A person invests $7650 in railroad stock, at 63¾. What will he receive if a dividend of 3¼% be declared?

Ex. If a person buys 6% stock at 120, what rate of interest does he receive on his money invested?

$100 stock costs $120. $100 stock pays $6.
Hence, the $120 invested pays $6.
Therefore, the rate of interest is 6 ÷ 120 = .05 or 5%.

25. If 3% stocks are at 88⅛, what rate per cent interest will a purchaser receive on his money?

26. If an 8% stock is at 150, what rate per cent interest will a purchaser receive on his money?

27. If a 10% stock is at 175, what rate per cent interest will an investor receive on his money?

28. If a 4½% stock is at 85, what rate per cent interest will a purchaser receive on his money?

29. If a 7% stock is at 114, what rate per cent interest will a purchaser receive on his money?

30. If a 6% stock is at 130, what rate per cent interest will a purchaser receive on his money?

31. If an 8% stock is at 140, what rate per cent interest will a purchaser receive on his money?

32. How much money must be invested in 4% stock, at 92, to receive $245 income?

33. Find the sum required for an investment in a 4% stock, at 98½, to produce an income of $200 a year.

34. A person bought some bank stock at 107, and received $384.25 when a dividend of 7¼% was paid. How much had he invested?

35. What must be the price of a 5% stock, in order that a buyer may receive 6% on his money?

CHAPTER XVIII.

POWERS AND ROOTS.

372. The *square* of a number is the product of *two* factors, each equal to this number.

Thus, the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

373. The **square root** of a number is one of the *two equal factors* of the number.

Thus, the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

374. The square root of a number is indicated by the *radical sign* $\sqrt{}$, or by the fractional exponent $\frac{1}{2}$ written above and to the right of the number.

375. Since $35 = 30 + 5$, the square of 35 may be obtained as follows:

$$\begin{array}{l} 30 + 5 \\ \underline{30 + 5 \qquad\qquad} \\ 30^2 + \ (30 \times 5) \\ \underline{\qquad\quad (30 \times 5) + 5^2} \\ 30^2 + 2(30 \times 5) + 5^2 \end{array} \qquad\qquad \begin{array}{r} 30^2 = 900 \\ 2(30 \times 5) = 300 \\ 5^2 = \underline{25} \\ = 1225 \end{array}$$

376. Hence, since every number consisting of two or more figures may be regarded as composed of tens and units,

The square of a number will contain the square of the tens + twice the tens × the units + the square of the units.

SQUARE ROOT.

377. The first step in extracting the square root of a number is to mark off the figures in periods.

Since $1 = 1^2$, $100 = 10^2$, $10{,}000 = 100^2$, and so on, it is evident that the square root of any number between 1 and 100 lies between 1 and 10; of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any number expressed by *one* or *two* figures is a number of *one* figure; of any number expressed by *three* or *four* figures is a number of *two* figures; and so on.

If, therefore, an integral number be divided into periods of two figures each, from the right to the left, the number of figures in the root will be equal to the number of periods. The last period at the left may consist of only one figure.

Ex. Find the square root of 1225.

```
    12 25(35
     9
   65)3 25
      3 25
```

The first period, 12, contains the square of the tens' number of the root.

The greatest square in 12 is 9, and the square root of 9 is 3. Hence 3 is the tens' figure of the root.

The square of the tens is subtracted, and the remainder, 325, is twice the tens × the units + the square of the units. Twice the 3 tens is 6 tens, and 6 tens is contained in the 32 tens of the remainder 5 times. Hence 5 is the units' figure of the root. Since twice the tens × the units + the square of the units is equal to (twice the tens + the units) × the units, the 5 units are annexed to the 6 tens, and the result, 65, is multiplied by 5.

378. The same method will apply to numbers of more than two periods, by considering the part of the root already found as so many tens with respect to the next figure.

Ex. Extract the square root of 7890481.

```
     7 89 04 81(2809
     4
   48)3 89
      3 84
  5609)5 04 81
       5 04 81
```

When the third period, 04, is brought down, and the divisor, 56, formed, the next figure of the root is 0, because 56 is not contained in 50. The 0 is then placed both in the root and the divisor, and the next two figures, 81, are brought down.

379. If the square root of a number have decimal places, the number itself will have *twice* as many.

Thus, if 0.11 be the square root of some number, the number will be $(0.11)^2 = 0.11 \times 0.11 = 0.0121$.

Therefore, the number of *decimals* in every square number will be *even;* and the number of decimal places in the root will be *half* as many as in the given number itself.

Hence, if a given square number contain a decimal, and if it be divided into periods of two figures each, by beginning at the decimal point and marking toward the left for the integral number, and toward the right for the decimal, the number of periods to the *left* of the decimal point will show the number of *integral* places in the root, and the number of periods to the *right* will show the number of *decimal* places in the root.

If a number contain an *odd* number of decimal places, or if any number give a *remainder* when as many figures in the root have been obtained as the given number has periods, then its exact square root cannot be found. We may, however, approximate to the exact root as near as we please, by annexing ciphers and continuing the operation. The result will be a constantly varying succession of figures.

Ex. Extract to six places of decimals the square root of 19.

```
  19 00 00 00(4.358899
  16
83)3 00
   2 49
865) 51 00
     43 25
 8708)7 75 00
       6 96 64
   8716)78 360
        69 728
          8 6320
          7 8444
            78760
            78444
```

In this example, after finding four figures of the root, the other three are found by common division. The rule in such cases is, that one less than the number of figures already obtained may be found without error by division, the divisor to be employed being twice the part of the root already found.

380. The square root of a common fraction is found by extracting the square roots of the numerator and denominator. But, when the denominator is not a perfect square, it is best to reduce the fraction to a decimal and then extract the root.

Thus, the square root of $\frac{121}{361} = \frac{11}{19}$.

EXERCISE LXIV.

Find the square roots of:

1. 2916.
2. 7921.
3. 494,209.
4. 20,164.
5. 3,345,241.
6. 125457.64.
7. 47,320,641.
8. 21,609.
9. 53.7289.
10. 883.2784.
11. 1.97262025.
12. .0002090916.

13. 2. **14.** 5. **15.** .3. **16.** $3\frac{1}{4}$. **17.** $8\frac{5}{6}$. **18.** .9.

19. Find in yards the side of a square field containing 20 acres.

20. Find the side of a square the area of which is 150 sq. ft. 9 sq. in.

21. Find the side of a square the area of which is 8 sq. yds. 7 sq. ft. 73 sq. in.

Find to six places of decimals the square roots of:

22. $\frac{4}{9}$; $\frac{5}{9}$; $\frac{1}{2}$; $\frac{3}{5}$; $\frac{5}{7}$; $\frac{3}{4}$; $\frac{2}{3}$; $\frac{5}{6}$.

Sometimes the square root of a number may be most easily found by resolving the number into its prime factors, and taking the roots of the factors separately.

Thus, $1089 = 3^2 \times 11^2$.
Hence, $\sqrt{1089} = 3 \times 11 = 33$.

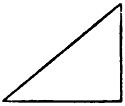
Find, by factoring, the square roots of:

23. 2025; 17.64; 2.0164; 533.61; 204.49.

381. *The sum of two numbers multiplied by their difference is equal to the difference of their squares.*

Thus, the sum of 3 and 5 is 8, their difference is 2, and $2 \times 8 = 16$; and the difference of their squares, $25 - 9$, is 16.

382. In a right triangle,



The sum of the squares on the two legs is equivalent to the square on the hypotenuse.

Thus, if 3 and 4 represent respectively the legs of a right triangle, the hypotenuse $= \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

And if 5 represent the hypotenuse of a right triangle, and 4 one leg, then the other leg is $\sqrt{(5+4)(5-4)} = \sqrt{9} = 3$.

24. A ladder 13 ft. long standing on level ground reaches a window 12 ft. from the ground. How far from the wall is the foot of the ladder?

25. The two legs of a right triangle are 35 in. and 84 in. respectively. Find the hypotenuse.

26. The hypotenuse of a right triangle is 61 in., and one leg 11 in. Find the other leg.

27. Find the longest straight line that can be drawn on the floor of a room 20 ft. by 15 ft.

28. Find the longest line in a box that is 8 ft. long, 4 ft. wide, 1 ft. deep.

CUBE ROOT.

383. The *cube* of a number is the product of *three* factors, each equal to the number.

The cubes of	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,
are	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000.

384. The **cube root** of a number is one of the *three equal factors* of the number.

Thus, the cube roots of	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000,
are	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.

385. The cube root of a number is indicated by $\sqrt[3]{\ }$, or by the fractional exponent $\frac{1}{3}$ written above and to the right of the number.

Thus, $\sqrt[3]{343}$, or $343^{\frac{1}{3}}$, means the cube root of 343.

386. Since $35 = 30 + 5$, the cube of 35 may be obtained thus:

$$\begin{array}{l}
30 + 5 \\
30 + 5 \\
\hline
30^2 + (30 \times 5) \\
\quad + (30 \times 5) + 5^2 \\
\hline
30^2 + 2(30 \times 5) + 5^2 \\
30 + 5 \\
\hline
30^3 + 2(30^2 \times 5) + (30 \times 5^2) \\
\qquad (30^2 \times 5) + 2(30 \times 5^2) + 5^3 \\
\hline
30^3 + 3(30^2 \times 5) + 3(30 \times 5^2) + 5^3
\end{array}
\qquad
\begin{array}{rr}
30^3 = & 27{,}000 \\
3(30^2 \times 5) = & 13{,}500 \\
3(30 \times 5^2) = & 2{,}250 \\
5^3 = & 125 \\
\hline
 & 42{,}875
\end{array}$$

Hence the cube of any number composed of tens and units contains four parts:

I. *The cube of the tens.*

II. *Three times the product of the square of the tens by the units.*

III. *Three times the product of the tens by the square of the units.*

IV. *The cube of the units.*

Since $1 = 1^3$, $1000 = 10^3$, $1{,}000{,}000 = 100^3$, and so on, it follows that the cube root of any number between 1 and 1000, that is, of any number that has *one*, *two*, or *three* figures, is a number of *one* figure; and that the cube root of any number between 1000 and 1,000,000, that is, of any number that has *four*, *five*, or *six* figures, is a number of *two* figures; and so on.

If, therefore, an integral number be divided into periods of three figures each, from right to left, the number of figures in the root will be equal to the number of periods. The last period may consist of one, two, or three figures.

Ex. Extract the cube root of 42875.

$$\begin{array}{rr|l} & & 42\,875(35 \\ & & 27 \\ \cline{3-3} 3 \times 30^2 = & 2700 & 15\,875 \\ 3 \times (30 \times 5) = & 450 & \\ 5^2 = & 25 & \\ \cline{2-2} & 3175 & 15\,875 \\ \cline{3-3} \end{array}$$

Since 42875 consists of two periods, the cube root will consist of two figures.

The first period, 42, contains the cube of the tens' number of the root.

The greatest cube in 42 is 27, and the cube root of 27 is 3. Hence 3 is the tens' figure of the root.

The remainder, 15875, resulting from subtracting the cube of the tens, will contain three times the product of the square of the tens by the units + three times the product of the tens by the square of the units + the cube of the units.

Each of these three parts contains the units' number as a factor.

Hence the 15875 consists of two factors, one of which is the units' number of the root; and the other factor is three times the square of the tens + three times the product of the tens by the units + the square of the units. The larger part of this second factor is three times the square of the tens.

And, if the 158 hundreds of the remainder be divided by the $3 \times 30^2 = 27$ hundreds, the quotient will be the units' number of the root.

The second factor can now be completed by adding to the 2700 $3 \times (30 \times 5) = 450$ and $5^2 = 25$.

Ex. Extract the cube root of 57512456.

$$\begin{array}{rr|l} & & 57\,512\,456(386 \\ & & 27 \\ \cline{3-3} 3 \times 30^2 = & 2700 & 30\,512 \\ 3 \times (30 \times 8) = & 720 & \\ 8^2 = & 64 & \\ \cline{2-2} & 3484 & 27\,872 \\ \cline{3-3} & & \quad 2\,640\,456 \\ 3 \times 380^2 = & 433200 & \\ 3 \times (380 \times 6) = & 6840 & \\ 6^2 = & 36 & \\ \cline{2-2} & 440076 & \quad 2\,640\,456 \\ \cline{3-3} \end{array}$$

387. If the cube root of a number have decimal places, the number itself will have *three times* as many.

Thus, if 0.11 be the cube root of a number, the number is $0.11 \times 0.11 \times 0.11 = 0.001331$. Hence, if a given number contain a decimal, and if it be divided into periods of three figures each, by beginning at the decimal point and marking toward the left for the integral number and toward the right for the decimal, the number of periods toward the *left* from the decimal point will show the number of *integral* places in the root, and the number of periods toward the *right* will show the number of decimal places in the root.

388. If the given number be not a perfect cube, ciphers may be annexed, and a value of the root may be found as near to the *true* value as we please.

Ex. Extract the cube root of 1250.6894.

$$\begin{array}{rrl} & & 1\,250.689\,400(10.77 \\ & & 1 \\ 3 \times 10^2 = & 300 & \overline{250} \end{array}$$

Since 300 is not contained in 200, the next figure of the root will be 0.

$$\begin{array}{rr|l}
3 \times 100^2 = & 30000 & 250\,689 \\
3 \times (100 \times 7) = & 2100 & \\
7^2 = & \underline{49} & \\
 & 32149 & \underline{225\,043} \\
 & & 25\,646\,400 \\
3 \times 1070^2 = & 3434700 & \\
3 \times (1070 \times 7) = & 22470 & \\
7^2 = & \underline{49} & \\
 & 3457219 & \underline{24\,200\,533} \\
 & & 1\,445\,867
\end{array}$$

It will be seen from the periods that the root will have two integral places, and therefore the decimal point must be placed in the root as soon as two figures of the root are obtained.

389. The following method very much shortens the work in long examples.

Ex. Extract the cube root of 5 to five places of decimals.

$$\begin{array}{rr|l}
 & & 5.000(1.70997 \\
 & & 1 \\ \hline
3\times 10^2 = & 300 & 4\,000 \\
3(10\times 7) = & 210 & \\
7^2 = & \left.\begin{array}{r}\underline{49}\\ 559\\ \underline{259}\end{array}\right\} & \begin{array}{l} \\ \underline{3\,913}\\ 87\,000\,000\end{array} \\
3\times 1700^2 = & 8670000 & \\
3(1700\times 9) = & 45900 & \\
9^2 = & \left.\begin{array}{r}\underline{81}\\ 8715981\\ \underline{45981}\end{array}\right\} & \begin{array}{l} \\ \underline{78\,443\,829}\\ 8\,556\,1710\end{array} \\
3\times 1709^2 = & 8762043 & \underline{7\,885\,8387} \\
 & & 670\,33230 \\
 & & \underline{613\,34301}
\end{array}$$

After the first two figures of the root are found, the next trial divisor is obtained by bringing down the sum of the 210 and 49 obtained in completing the preceding divisor, then adding the three lines connected by the brace, and annexing two ciphers to the result.

It is seen at a glance that, when the trial divisor is increased by 3 times the 17 tens of the root, it will be greater than 87000; so that 0 is placed in the root, and 3×1700^2 is obtained by annexing two ciphers to the 86700. Again the trial divisor is obtained by bringing down the sum of the 45900 and 81, which was obtained in completing the preceding divisor, then adding the three lines connected by the brace, and annexing two ciphers to the result.

The last two figures of the root are found by division. The rule in such cases is, that two less than the number of figures already obtained may be found without error by division, the divisor to be employed being three times the square of the part of the root already found.

390. The cube root of a common fraction is found by taking the cube roots of the numerator and denominator; but, if the denominator be not a perfect cube, it is best to reduce the fraction to a decimal, and then extract the root.

EXERCISE LXV.

Find the cube roots of:

1. 1331.
2. 1728.
3. 12.167.
4. 300.763.
5. 148877.
6. 2048383.
7. 59.776471.
8. 304957.115891.
9. .007821346625.
10. 104.600290750613.
11. 17183498535125.
12. 122615.327232.
13. 10; $3\frac{5}{8}$; $8\frac{1}{8}$ to four places of decimals.
14. 5; $\frac{5}{9}$; $7\frac{3}{5}$; $\frac{3}{4}$ to four places of decimals.
15. Find the entire surface of a cube the volume of which is 14 cu. ft. 705.088 cu. in.

The square of $(30 + 5) = 30^2 + 2(30 \times 5) + 5^2$. § 375.

The 30^2 may be represented by a square (Fig. 1) 30 in. on a side.

The $2(30 \times 5)$ may be represented by two strips 30 in. long and 5 in. wide, of Fig. 2, which are added to two adjacent sides of Fig. 1.

The 5^2 may be represented by the small square of Fig. 3 required to make Fig. 2 a complete square.

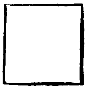


Fig. 1.

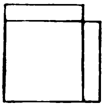


Fig. 2.

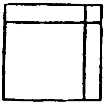


Fig. 3.

In extracting the square root of 1225, the large square, which is 30 in. on a side, is first removed, and a surface of 325 sq. in. remains.

This surface consists of two equal rectangles, each 30 in. long, and a small square whose side is equal to the width of the rectangles.

The width of the rectangles is found by dividing the 325 sq. in. by the sum of their lengths, that is, by 60, which gives 5 in.

Hence the entire length of the surfaces added is 30 in. + 30 in. + 5 in. = 65 in., and the width is 5 in.

Therefore the total area is $(65 \times 5) = 325$ sq. in.

The cube of $(30 + 5) = 30^3 + 3(30^2 \times 5) + 3(30 \times 5^2) + 5^3$. § 386.

The 30^3 may be represented by a cube whose edge is 30 in. (Fig. 1).

The $3(30^2 \times 5)$ may be represented by three rectangular solids, each 30 in. long, 30 in. wide, and 5 in. thick, to be added to three adjacent faces of Fig. 1.

The $3(30 \times 5^2)$ may be represented by three equal rectangular solids, 30 in. long, 5 in. wide, and 5 in. thick, to be added to Fig. 2.

The 5^3 may be represented by the small cube required to complete the cube of Fig. 3.

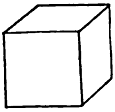


Fig. 1.

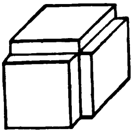


Fig. 2.

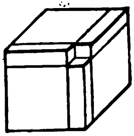


Fig. 3.

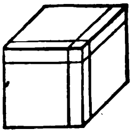


Fig. 4.

In extracting the cube root of 42875, the large cube (Fig. 1), whose edge is 30 in., is first removed.

There remain $(42875 - 27000)$ cu. in. = 15875 cu. in.

The greater part of this is contained in three rectangular solids which are to be added to Fig. 1, and which are each 30 in. long and 30 in. wide.

The thickness of these solids is found by dividing the 15875 cu. in. by the sum of the three faces, each of which is 30 in. square; that is, by 2700 sq. in. The result is 5 in.

There are also three rectangular solids which are to be added to Fig. 2, and which are 30 in. long and 5 in. wide; and a cube which is to be added to Fig. 3, and which is 5 in. long and 5 in. wide.

Hence the sum of the products of two dimensions of all these solids is

For the larger rectangular solids, $3(30 \times 30)$ sq. in. = 2700 sq. in.
For the smaller rectangular solids, $3(30 \times 5)$ sq. in. = 450 sq. in.
For the small cube, (5×5) sq. in. = 25 sq. in.
3175 sq. in.

This number multiplied by the third dimension gives (5×3175) cu. in. = 15,875 cu. in.

391. In bodies of the same shape,

Two corresponding lines are in the same ratio as any other two.

The ratio of two corresponding surfaces is the square of the ratio of two corresponding lines.

The ratio of two corresponding volumes is the cube of the ratio of two corresponding lines.

Conversely,

The ratio of two corresponding lines is the square root of the ratio of two corresponding surfaces, and the cube root of the ratio of two corresponding volumes.

EXERCISE LXVI.

1. If the diameter of the moon be reckoned at 2000 mi., and that of the earth at 8000 mi., find the ratio of the surface of the moon to that of the earth. Also, find the ratio of the volume of the moon to that of the earth.
2. If the diameter of the earth be reckoned at 8000 mi., that of Jupiter at 84,000 mi., and that of the sun at 880,000 mi., find the ratios of their volumes.
3. If the diameters of two circles be 20 in. and 40 in. respectively, find the ratio of their circumferences and the ratio of their surfaces.
4. If the areas of two circles be 8000 sq. in. and 36,000 sq. in. respectively, find the ratio of their diameters to the nearest thousandth of an inch.
5. If the volumes of two spheres be 100 cu. in. and 1000 cu. in. respectively, find the ratio of their diameters to the nearest thousandth of an inch.
6. If two stacks of hay of the same shape contain 4 t. 6 cwt. and 1 t. 8 cwt. respectively, find the ratio of their heights.

7. If an ox 7 ft. in girth weigh 1500 lbs., what will be the girth of a similar ox weighing 2500 lbs.?

8. The surface of a pyramid is 560 sq. in. What is the surface of a similar pyramid whose volume is 27 times as great?

9. The volume of a pyramid is 1331 cu. in. What is the volume of a similar pyramid whose surface is 4 times as great?

10. If a well-proportioned man 5 ft. 10 in. high weigh 160 lbs., what should a man 6 ft. high weigh, to the nearest tenth of a pound? What should be the height, to the nearest tenth of an inch, of a man weighing 210 lbs.?

11. A three-gallon jug and a one-gallon jug are of the same shape. What, to the nearest thousandth, is the ratio of their diameters?

12. Two hills have exactly the same shape; one is 900 ft. high, the other 1200 ft. Find the ratio of their surfaces, and also the ratio of their volumes.

13. A ball 3 in. in diameter weighs 4 lbs.; another ball of the same metal weighs 9 lbs. Find the diameter of the second ball to the nearest thousandth of an inch.

14. If Apollo's altar were a perfect cube 10 ft. on a side, what, to the nearest hundredth of an inch, would be the dimensions of a new cubical altar containing twice as much stone?

15. A man standing 40 ft. from a building 24 ft. wide observed that, when he closed one eye, the width of the building hid from view 90 rods of fence which was parallel to the width of the building. Find the distance from the eye of the observer to the fence.

16. A bushel measure and a peck measure are of the same shape. Find the ratio of their heights.

CHAPTER XIX.

MISCELLANEOUS PROBLEMS.

AVERAGES.

1. If a dozen eggs weigh 692^g, what is the mean weight of an egg?

2. Seven boys weigh respectively 119.7 lbs., 105 lbs., 178.3 lbs., 165.3 lbs., 142.8 lbs., 109 lbs., 154.2 lbs. What is their average weight?

3. A merchant mixes 1 lb. of coffee worth 27 cents, 1 lb. worth 35 cents, and 1 lb. worth 40 cents. What are the three pounds together worth? How much a pound is the mixture worth?

4. A merchant mixes 2 lbs. of coffee worth 27 cents a pound, 3 lbs. worth 35 cents a pound, and 1 lb. worth 40 cents. What is a pound of the mixture worth?

5. What is the value per pound of a mixture of coffee containing 7 lbs. worth 26 cents a pound, 4 lbs. worth 31 cents a pound, and 10 lbs. worth 34 cents a pound?

6. What is the cost of a gallon of the mixture in which 7 gals. cost 67 cents a gallon, 5 gals. cost 48 cents a gallon, and water, without cost, was added until there were 15 gals. of the mixture?

7. If 7^l of water be poured into a vessel containing 3^l of sulphuric acid, specific gravity 1.840, and the mixture shrink to 9.972^l, what is the specific gravity of the mixture?

8. If 4^l of water and 1^l of sulphuric acid, specific gravity 1.842, when mixed shrink $\frac{1}{4}$ of 1%, what is the specific gravity of the mixture?

Ex. In what proportion must alcohol of specific gravity .800 be mixed with water to bring the specific gravity to .925, if no allowance be made for condensation?

The specific gravity of the alcohol lacks .125 of the required specific gravity; and specific gravity of the water is .075 above the required specific gravity.

Therefore, the alcohol is to the water in the inverse ratio of 125 to 75. That is,

$$\text{Alcohol} : \text{water} = 75 : 125 = 3 : 5.$$

To test this answer: 3^l of alcohol weigh 2.4^{kg}, 5^l of water weigh 5^{kg}; therefore, 8^l of the mixture weigh 7.4^{kg}, and the specific gravity of the mixture is $\frac{7.4}{8} = .925$.

9. In what proportions must tin of specific gravity 7.29 and lead of specific gravity 11.35 be mixed to make a solder of specific gravity 10.21, if no allowance be made for expansion or condensation? (Test the answer.)

10. In what proportions must oils worth $1.25 and 80 cents a gallon be mixed to make a mixture worth $1 a gallon? (Test the answer.)

(1) Find the cost of the following draft, exchange being $\frac{1}{2}$ of 1% premium, and interest 6%.

$800. CINCINNATI, O., Feb. 8, 1881.

Thirty days after sight pay to the order of S. Clark eight hundred dollars, and place to the account of

To H. WRIGHT, Philadelphia. P. CLEMENT.

	$800.00	
.0055 of $800 =	4.40	discount at 6% for 33 days.
	$795.60	proceeds of draft (cost at par).
.005 of $800 =	4.00	premium at ½ of 1%.
	$799.60	cost of draft.

11. Find the cost of a draft on New York for $1100, at ¼ of 1% premium.

12. Find the cost of a draft on New Orleans for $1350, at ¼ of 1% discount.

13. Find the cost of a draft for $1600, payable 30 days after sight, when exchange is ¼ of 1% premium, and interest 6%.

14. Find the cost of a draft for $500, payable 60 days after sight, when exchange is ½ of 1% discount, and interest 7%.

15. Find the cost of a draft for $1200, payable in 90 days after sight, when exchange is ½ of 1% premium, and interest 7%.

16. Find the cost of a draft for $950, payable in 30 days, when exchange is at par, and interest 4½%.

17. Find the cost of a draft for $725, payable in 60 days, when exchange is at ¼ of 1% discount, and interest 5%.

18. Find the cost of a draft for $810, payable in 90 days, when exchange is at ¼ of 1% premium, and interest 5½%.

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19. Make six different numbers with the digits 1, 2, 3, and find their sum.

20. Show, without division, that 36,432 contains 8, 9, 11 as factors.

21. The number of seamen at Portsmouth is 800, at Charlestown 404, and at Brooklyn 756. A ship is commissioned whose complement is 490 seamen. Determine the number to be drafted from each place in order to obtain a proportionate number from each.

22. Find the smallest multiplier that will make 47,250 a perfect cube.

23. Find the surface of a sphere 18.3 in. in diameter.

NOTE. The area of a circle is 3.1416 times the square of the radius; and the surface of a sphere is 4 times the area of a circle of the same radius as the sphere.

24. What is the horizontal distance between two points, when the air-line distance is 1534 ft., and the difference of level 34 ft.?

25. Find the horizontal distance when the road distance is 1 mile, and the rise 347 ft.

26. If the road distance is half a mile, and the horizontal distance 2513 ft., find the difference of level.

27. The diagonal of a rectangular floor is 34.6 ft., and the width is 17.8 ft. Find the length of the floor.

28. The height of a tower on a river's bank is 55 ft., the length of a line from the top to the opposite bank is 78 ft. Find the breadth of the river.

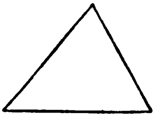
29. The population of a city in 1880 was 12,298, showing a decrease of 8⅓% on its population in 1870; in 1870 there was an increase of 7½% on the census of 1860. What was its population in 1860?

30. Find the increase of income obtained by transferring $2500 from 3% stocks at 94½ to 4% stocks at 105.

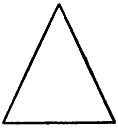
31. Each person breathing in a closed room spoils the air at the rate of about 8 cu. ft. a minute. A congregation of 400 persons enter a closed room 70 ft. by 40 ft. and 20 ft. high. How long will it take them to spoil the air?

32. How long can the windows and doors of a school-room be safely kept closed when occupied by 50 children, if the room is 25 ft. by 20 ft. and 10 ft. high?

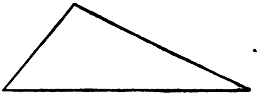
392. When the three sides of a triangle are known, the area is found as follows:



I.



II.



III.

Subtract each side separately from half the sum of the three sides.

Find the continued product of the half-sum and the three remainders, and extract the square root of that product.

Ex. Find the area of a triangle whose sides are 3, 4, and 5 ft. respectively.

The half-sum of the sides is $\frac{3+4+5}{2} = 6$ ft., and the three remainders are 3 ft., 2 ft., 1 ft.

The area is $\sqrt{6 \times 3 \times 2 \times 1}$ sq. ft. $=$ 6 sq. ft.

33. Find the area of a triangle whose sides are 73, 57, and 48 ft.

34. Find the number of hektars in a triangular field whose sides are 37.5^{m}, 91.7^{m}, and 78.9^{m}.

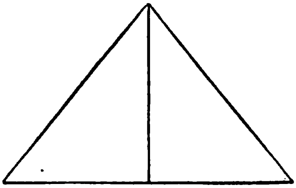
35. Find the number of acres in a triangular field whose sides are 227, 342, and 416 ft.

36. Find the number of acres in a triangular field whose sides are 79 chains 8 links, 57 chains 3 links, and 102 chains 19 links.

37. Find the number of square rods in a triangle whose sides are 7 rds. 2 yds., 6 rds. 5 yds., and 9 rds. 4½ ft.

38. Find the number of acres in a four-sided field, the sides of which are in order 361, 561, 443, and 357 ft.; and the distance from the beginning of the first side to the end of the second side is 682 ft.

393. When one side of a triangle is regarded as a base, the triangle may be imagined as resting with that base on a horizontal line. The distance of the highest point of the triangle above that line is called the *altitude* of the triangle.



When the altitude and base of a triangle are known, the area is found as follows:

Multiply the altitude by the base, and take one-half the product.

Ex. Find the area of a triangle of which the base is 3 ft. and altitude 4 ft.

$$\left(\frac{3\times 4}{2}\right)\text{ sq. ft.} = 6\text{ sq. ft.}$$

39. Find the number of hektars in a field of three sides, one of which is 82.1^{m}, and the distance from this side to the opposite corner 47.3^{m}.

40. Find the number of acres in a circular field 213 yds. 2 ft. across.

41. Find the number of acres in a triangular lot, one side of which is 343.6 ft., and the distance from this side to the opposite corner is 163.2 ft.

When the three sides of a triangle are known, and the altitude is required:

Find the area of the triangle, and divide the result by half the side that is taken as the base.

42. Find the altitude of a triangle, if each side is 1000 ft.

43. Find the distances of the vertices from the opposite sides of a triangle, when these sides are 17.8^{mm}, 23.6^{mm}, and 31.5^{mm}.

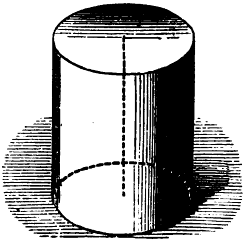
44. If the four sides of a field measured in succession are 237, 253, 244, and 261 ft., and the diagonal measured from the end of the first side to the end of the third side is 351 ft.; find its area.

45. Find the square factors contained in 33,075.

46. The top of St. Peter's, Rome, is $\frac{9}{110}$ of a mile above the ground, and that of St. Paul's, London, is $\frac{17}{264}$ of a mile. By how many feet does the height of St. Peter's exceed that of St. Paul's?

47. How many days elapsed between the annular eclipse of May 15, 1836, and that of March 15, 1858?

394. A straight, round stick, cut off square at each end, is called a **cylinder.**



Cylinder.

The area of the convex surface of a cylinder is obtained as follows:

Multiply the circumference of one end by the length of the cylinder.

The *volume* of a cylinder is obtained as follows:

Multiply the area of one end by the length of the cylinder.

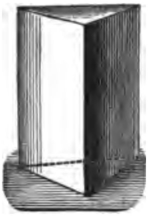
48. Given a cylinder 10 in. in diameter and 12 in. long; required the area of each end, the convex area, the total area, and the contents in gallons.

49. Find the capacity in gallons of a round cistern 13 ft. in diameter and 9 ft. deep.

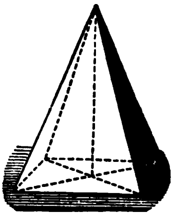
50. What must be the diameter of a cylinder 10 in. deep, in order that it may hold 1 gallon?

395. A solid with two equal polygonal ends, connected by plane faces at right angles to the ends, is called a **prism.** The volume of a prism is found as follows:

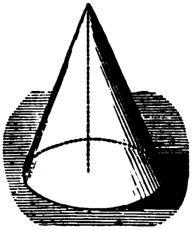
Multiply the area of one end by the length of the prism.



Prism.



Pyramid.



Cone.

51. Find the volume of a triangular prism 11 in. long, the sides of the ends being 2, 3, and 4 in. long.

52. Find the capacity in bushels of a bin 6 ft. long, and the end of which is a square measuring 3 ft. 3 in. on a side.

53. Find the number of cubic yards in a square prism 200 ft. on a side, and 40 ft. long.

396. A solid with a polygonal base, and plane faces meeting in a point, is a **pyramid.** The volume of a pyramid is one-third of that of a prism of the same base and height.

54. How many cubic yards in a square pyramid 210 ft. on a side, and 123 ft. high?

397. A body whose base is a circle, and whose convex surface tapers uniformly to a point, is called a **cone.**

The volume of a cone is one-third the volume of a cylinder of the same base and height.

55. The slant depth of a conically-shaped drinking-cup is 93^{mm}, and the diameter at the top 8^{cm}. What is its capacity?

56. The volume of a cone is 1^{cbm}; its height is equal to the radius of its base. Find the dimensions of the cone.

398. The capacity of a round vessel, that is not hemispherical, cylindrical, or conical, may be estimated as follows:

Add one-fourth of the square of the diameter to one-third of the square of the depth, and multiply the result by eleven-sevenths of the depth.

57. Find the capacity of a wash-bowl 30^{cm} in diameter and 5^{cm} deep.

58. Find the capacity in liters of a boiler 89^{cm} in diameter and 31^{cm} deep.

59. Find the capacity in quarts of a bowl 10 in. in diameter and 4 in. deep.

60. Find the capacity in pints of a saucer 6 in. across and 1½ in. deep; of a bowl 7 in. across and 3 in. deep; of a bowl 8 in. across and 3½ in. deep.

61. How many gallons will a boiler 30 in. in diameter and 1 ft. deep hold.

399. The oval made by the shadow of a circular plate is called an **ellipse.**

The area of an ellipse is .7854 *of the product of its longest and shortest diameters.*

62. Find the area of an ellipse 8 in. by 11 in.; of an ellipse 15 in. by 21 in.

63. A cylindrical log, 11 in. in diameter, is sawed off on such a slant that the pieces are 8 in. longer on the longest than on the shortest side. Find the dimensions of the ellipse thus made, and its area.

400. *The number of vibrations that pendulums make in a given time is inversely as the square root of their lengths.*

A pendulum passing its central point of rest once every mean solar second is 39.138 in. long.

64. Find the length of a pendulum beating half-seconds; of a pendulum beating quarter-seconds.

65. If a cannon-ball be suspended by a fine wire 176 ft. long in the central well of the Bunker Hill Monument, how many times a minute will it swing?

401. *If a plunger fits tightly in a small cylinder, and by it water is forced into a large cylinder, the plunger in the large cylinder is lifted with a force nearly equal to the product of the force with which the little plunger is driven in multiplied by the square of the ratio of the diameters of the two cylinders.*

66. Find the lifting-power of a hydraulic press, the plunger being 1^{cm} in diameter and driven with a force of 100^{kg}, if the lifting-piston is 1^{m} in diameter.

67. If the plunger is ½ in. in diameter, and is driven with a force of 1000 lbs., how much can it lift with a lifting-piston 4 ft. in diameter?

402. *When a body is moving in a circle, the centrifugal force is about* 1.227 *of the continued product of the weight of the body, the number of feet in the radius of the circle, and the square of the number of revolutions in a second.*

Thus, a body going round a circle of 5 ft. radius once a minute presses away from the centre with a force equal to $1.227 \times 5 \times \frac{1}{60^2}$ of the weight of the body.

NOTE. When the radius is measured in meters, the multiplier 4.025 must be used in place of 1.227.

68. If a top 3 in. in diameter is making 200 revolutions a second, with what force does the outer layer pull away from the centre?

69. If a sling 30 in. long contains a stone, and is whirled round 80 times a minute, what is the force pulling on the string?

403. A body falling in a vacuum falls 4.903^{m} in the first second; it then has acquired a velocity of 9.806^{m}.

A falling body increases its velocity in proportion to the time it is falling; and the distance fallen is in proportion to the square of the number of seconds of time it is falling.

Thus, a body falling from a state of rest in a vacuum will in half a second have fallen 1.2257^{m}, and have acquired a velocity of 4.903^{m}; in 3 sec. it will have fallen 44.127^{m}, and have acquired a velocity of 29.418^{m} per second.

The velocity of heavy bodies falling short distances in air will not be much less.

70. What velocity in meters a second will a cannon-ball acquire in falling three-quarters of a second? in falling three and a quarter seconds?

404. In the average state of the atmosphere, the distance at which an object is visible at sea is found by the following formula:

The square of the distance in English miles is seven-fourths the height of the object in English feet.

71. A hill 482 ft. high is 8 mi. from the shore. How many miles out at sea is it visible?

72. A sailor at the topmast 80 ft. above the sea can just see a sailor at the topmast of a similar ship. How many miles apart are the vessels?

73. If Mount Washington is 6240 ft. high and 76 mi. in an air-line from Cape Elizabeth, how far out from the Cape will its peak be visible in the ordinary state of the atmosphere?

405. When the distance is given in kilometers, and the height in meters,

The square of the distance in kilometers is 15 *times the height in meters.*

74. If in steaming away from a mountainous island a sailor estimates his distance at 171^{km}, when the island disappears beneath the wave, how high shall he estimate the mountains?

406. Sound travels in still air at 57° F. 1114 ft. a second, and 1 ft. a second faster for every degree above 57; so that at 63° it goes 1120 ft. a second; at 53°, 1110 ft. a second.

75. The flash of a gun is seen 7½ sec. before the report of the gun is heard; there is no wind, and the thermometer is 73° F. How far off was the gun?

76. A meteor was seen to burst; the report followed in 4 min. 17 sec. What was its distance if the average temperature of the intervening air was 50° F.

77. Find the weight supported by each of two men, A and B, who carry a hundred-weight suspended on a pole 6 ft. long, at a point 2 ft. 3 in. from A's end, if the pole weighs 16 lbs.

Note. One-half the pole is supported by each man, and the part of the weight supported by each is inversely proportional to its distance from him.

78. A man who rows 4 mi. an hour in still water takes 1 hr. 12 min. to row the same distance up a river. How long will it take him to row down again?

79. A man whose weight is 160 lbs., wishing to raise a rock, leans with his whole weight on a horizontal crowbar 5 ft. long, which is propped at the distance of 4 in. from the end in contact with the rock. Find what force he exerts on the rock, and what pressure the prop has to sustain, if the weight of the crowbar is not reckoned.

$$\frac{\text{Power}}{\text{Weight}} = \frac{\text{Distance of weight from fulcrum}}{\text{Distance of power from fulcrum}}.$$

80. A child weighing 56 lbs. is at one end of a plank 16 ft. long, and a child weighing 72 lbs. is at the other end. Find the distance of each child from the fulcrum when the plank is used for a see-saw.

81. A cask of lamp-black weighing 160^{kg} is attached to a rope wound on an axle 19^{cm} in diameter; at one end of the axle is a wheel 175^{cm} in diameter. With what force must a man pull down on a rope passing over the wheel to raise the cask if 1¾% of the pull is required to overcome friction?

$$\frac{\text{Force}}{\text{Weight}} = \frac{\text{Diameter of axle}}{\text{Diameter of wheel}}.$$

ANSWERS.

Art.
45. 995;
398.8;
488.92;
1689.91.
46. 1707.8;
3271.1;
1945.66;
3440.4;
2018.44;
8440.75.
47. 2372.64;
8794.95;
3625.3;
2299.86;
2062;
1897.8236;
1900.01828.
48. 6.64528;
30,106.937;
2284.57828;
371.8273.
49. 848.87;
1748.87;
3041.87;
959.87;
1000.87;

Art.
49. 1110.87;
1000;
1299.07;
770.22;
1342.17.
50. 886.353;
950.998;
347.0759;
724.8363;
1694.6084.
52. 6.6452727;
7.7847491;
10.6313239;
20.8266782;
1.2272041;
1.2099648.
53. 1.30103;
4.5861528;
3.5282304;
19.0826303;
16.5414467;
6.0157063;
4.2070434;
5.2839031;
3.2384788;
8.485051;

Art.
53. 1.2626906;
6.3834733.
54. 7.603166;
11.4093504;
6.7286717;
10.528675;
3.2657244;
21.712055;
19.9922991;
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20.0826303.
55. 4.523467;
14.8237261;
22.1157111;
24.8583194;
10.2590772;
29.6490831.
57. 1448.95;
2393.60;
1870.27;
2956.89;
4267.58;
3950.41;
6570.39;
3501.93;
4064.96.

Art.
69. 3; 3;
7; 3;
3; 9;
14;
22;
22.
70. 40 mi. N.;
$4.
73. 111;
220;
301;
432;
553;
602;
716;
876;
877;
9978;
4997.
74. 333;
423;
522;
5922;
6399;
6397;
7341;
6550;
3086;
3878;
9334;
2999.
75. 195;
119;
1822;

Art.
75. 1802;
314;
51;
118;
2482;
15,545;
1388.
76. $106.98;
$87.69;
$97.87;
$141.73;
$23.50;
$118.46;
$307.92;
$26.38.
77. 7066;
6975;
7287;
2848;
1747;
7286;
6987;
6960;
7391;
3251.
78. .4233109;
.2617994;
4.3761829;
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1.6715398;
3.1636232;

Art.
78. 15.1683487;
.3178372;
1.854102;
.317004;
.5040172;
1.618034;
.080936.
79. 2.3561945;
1.5707963;
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81. 46.
82. 19,418.
83. 382.
84. 46.
85. 329.426.
86. 4615.71.
87. 115,946.
88. 182,741.
89. 3822.161.
90. 1,107,790.
91. $501.02.
92. $72.17.
93. $235.56;
$180.37.
94. 66 yrs.;
45 yrs., 11 mos., 30 days.

Art.
95. 73;
115;
128;
116;
264;
284;
297;
320;
354;
369.
96. 58, (59);
89, (90);
88, (89);
241, (242);
180, (181);
121, (121).
97. 1, 2;
2, 3;
3, 5;
4, 4;
2, 6;
1, 7.
98. 6689;
8334.68.
99. 7.9585202.
100. 0.80415828.
101. 609 gals.
102. 12;
17;
13.37;
15.8958;
100.
103. 0.5;

Art.
103. 47;
.468;
.4764;
.4764012.
104. 3005;
2494;
1997;
1484;
428;
939;
1436;
1949.
105. 31,385.
106. $3.15.
107. $8138.25;
$7245.75.
108. A.D. 899;
A.D. 1087;
B.C. 399;
B.C. 347;
B.C. 382;
A.D. 70.
109. James, 1 more.
116. 555;
369;
462;
342;
1684;
2560;
17,312;
7122;
14,256;
58.009;
28.8594;

Art.
116. 439.726;
74.2064;
469.206.
117. 3.1415928;
3.1415928;
78.53981625.
118. 34.5576;
37.6992;
40.8408;
43.9824;
47.124;
24.596748;
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120. 31.4159265;
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122. .236;
.0236;
.00063;
.078;
.00431;
.002331;

Art.
122. .0065;
.00012.
123. $98.10;
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$608.40;
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$1028.40.
124. $0.83625;
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129. 1.570796;
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235.6194;
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672.300688;
67.2300688;
.0672300688;

Art.
129. 7.85398;
786.183398;
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749.269692;
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130. 215.042;
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161.2815;
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131. .98994952;
.9998490152;
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1.979894;
1.999975782;
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2.999824;
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133. 63,231.52;
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134. 2,914,700;
278,560;
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7854.
135. 1584;
3696;
897.6;
3025.44;

Art.
135. 4466.88.
138. 7.77546;
.777546;
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52.20292;
5.167932;
82.8089;
8532.7315.
139. 90.85041;
230.52519;
4.6723068;
5.5190079;
6.1370379;
3.136364;
4.183564;
4.7108292;
366.509528.
140. 4853772;
.3000228;
.61685316;
.237908;
1.679724;
1.038276;
2.1347172;
1.649826.
141. 34.109316;
20.103747;
2.296;
4.74165;
11.647195;
.110999;
8.362134;
74.538.
144. 7.02460;

Art.
144. 20,000,001.06;
.381966;
5000.000;
2999.99.
145. 1. $626;
$624;
$628;
$626.
2. 1346.
3. 16,011 bu.
4. $52,976,000.
5. 767,340.
6. 252,321.5.
7. $73.20.
8. $43.42.
148. 1. 7^3;
8^3;
2^9;
5^6.
2. 3.01^3;
$.67^{10}$;
$.208^4$;
3. 2.003^6;
20.03^4;
20.03^3.
153. 321;
423;
182;
141;
212;
10.41;
12.004;
1.21;
.144.

Art.
158. **1.** .04285;
.000027;
36,666.66666.
2. 23.33333;
1571.42857;
18.57142.
3. 24.28571;
.00063;
.01181.
4. .01545;
.05833;
.0025.
5. 91.66666;
1.41666;
14.16666.
6. 2266.66666;
1133.33333;
.12363.
7. 320;
32;
3.2.
8. .032;
1.17777;
.00011.
160. **1.** 3.35977;
1.75499;
94.66644;
184.98040.
2. 1.76470;
1.73410;
1.73210;
1.73205.
3. .0000254.
4. 1.41423;

Art.
160. **4.** 2.23613.
165. **1.** 2.
2. 684.
3. 1095.
4. 330.
5. 236.4.
6. 6048.
7. 6700.
8. 30.12.
9. .9072.
10. 44.1.
11. 215,240.
12. 6520.5.
13. 217.
14. 3040.
15. 259.
16. 324.
17. 22.5.
18. 33.75.
167. **1.** 2;
4;
5;
.5;
.4015;
.90202.
2. .70710.
3. .70710.
4. 3.14159;
.31830.
5. 54022;
.60919;
.5;
.33333.
6. $1\frac{184}{898}$;

Art.
167. **6.** $19\frac{180}{181}$;
$\frac{1}{8}$;
.5.
7. 2;
$2\frac{28}{171}$;
$\frac{1}{2}$;
.11257;
.01;
.00011.
8. .9144.
9. .8361.
10. 7645.
11. 1.1012.
12. 2.2046.
13. 3.1416.
14. .1722.
168. **1.** 1;
10;
100;
1000;
10,000;
100,000;
1,000,000.
2. 1;
$\frac{1}{10}$;
$\frac{1}{100}$;
$\frac{1}{1000}$;
$\frac{1}{10000}$;
$\frac{1}{100000}$;
$\frac{1}{1000000}$.
3. 1;
1;
1;
1; $\frac{1}{10}$.

Art.

168. **4.** 10,000; $\frac{1}{10000}$; 100; 100.

5. $\frac{1}{10}$; 10; $\frac{1}{1000000}$; 10.

6. 10; $\frac{1}{10}$; 10; 1.

7. 1.030301; 1.01; .99009.

Exercise I.

Page 56.

17. 73.603891.

18. 1426.8527.

19. 61.942.

20. 11.5111; 7.41917.

21. 19.0554; 21.8365.

22. 24.7183; .8531.

23. .45763; 99.99824.

24. 8.5125.

25. 24.9914.

26. 20.6445; .2412; .0224.

27. 10.30005; 241.46387; .01004.

28. .000006; 453.646; .43104.

29. .343; .03555.

30. 1.13925.

31. 9.0601; .002025; .00006561; 132.682214448064; .438976.

32. .000512.

33. 48.64; .01078; 8.8089.

34. 149.7; 3.593; 7557.

35. 10.932; .4575; 2760.

36. 39,840; 1000; 10,000,000.

37. .02294; 2.25; .1794.

38. 11,875; 2894.53125; 7.8125.

39. .10720; 20.41482; .17857.

40. .06905; 58,257.40365; 338,983.05084.

Exercise II.

Page 57.

1. 7.375.

2. 6.

3. 10.

4. 5.555.

5. 1.

6. 5.

7. 20.

8. 9.

9. .06.

10. .202272.

11. .0078.

12. .00112914.

13. 3.

Exercise III.

1. 909.
2. 42.
3. 935.
4. 2.27083 mi.
5. 3.4167 hrs.
6. 21.36288;
6.36618.
7. 96.248 in.
8. 336.13445.

Page 58.

9. 350.25 lbs.
10. 260,078.
11. 601.
12. $2184.91.
13. 4032; $3.24.
14. $5934.
15. 61.
16. 37.56097 min.;
49.67741 min.
17. 8857.98276 min.
18. 14,925.37313.
19. $31.45.
20. $196.80.
21. 66,662.97.

Page 59.

22. 8590.10270.
23. 7.48051;
9.30917;
1.24445;
3.38375.
24. $8383.67.
25. 7068.
26. 3 dys.;
6 dys.;
6 dys.
27. H., $0.54;
J., $0.90;
R., $0.36.
28. B., $41.48;
C., $46.36.
29. $63.20.
30. 420; 50.
31. 6000.
32. 24,780 lbs.;
25,620 lbs.

Page 60.

33. 708 lbs., dry;
732 lbs., wet.
34. 15.245 cu. in.;
113.34863 lbs.
35. 1.81357.
36. 7.49064.
37. 7.50334.
38. 11,190,000 mi.;
18,011,400^{km}.
39. 671,400,000 mi.;
1,080,684,000^{km}.
40. 449.43724.
41. 431.03448.
42. .2178.

Page 64.

1. 5.427^{km};
5,427,000^{mm};
542,700^{cm}.
2. 6.853^{m}; 685.3^{cm};
.006853^{km}.
3. 4970^{cm};
49,700^{mm};
.0497^{km}.
4. 1,240,000^{cm};
12,400,000^{mm}.
5. 1.23^{km};
123,000^{cm}.
6. 12.30^{m};
12,300^{mm}.
7. 172.718^{m}.

8. 862.05^{m}; $.00083^{m}$.
9. 365.6^{m}; 1.852272^{m}; $.364896^{m}$.
10. 391.94845^{m}; 16^{m}.
11. $11.59.
12. $45.
13. 42.05^{m}.
14. $3.12.
15. $0.014.
16. 533.33^{m}.
17. .07.
18. $147.

Page 66.

1. 185.4276^{ha}; 1.854276^{qkm}.
2. 278.56^{ha}.
3. $17,431^{qcm}$; $1,743,100^{qmm}$.
4. 174.675^{qkm}.
5. $1,361,400^{qm}$.
6. $22,500^{qm}$.

Page 67.

7. 137^{qcm}.
8. 357.1^{qmm}.

Page 68.

1. $2,250,000^{ccm}$.
2. 2.162875^{cbm}.

Page 69.

1. 1700^{l}; 157.854^{l}.
2. 9500^{ccm}; 15^{ccm}.
3. $125,000^{ccm}$; $.125^{cbm}$.
4. 4.3188^{hl}; $.43188^{cbm}$.
5. 375^{l}; $375,000^{ccm}$.
6. 734.159651^{l}; 7.34159651^{hl}; $.734159651^{cbm}$.
7. 8.573412867^{cbm}.
8. $734,578.912^{ccm}$; 734.578912^{l}.
9. 1.7315^{cbm}; $1,731,500^{ccm}$.

Page 70.

1. 1730; 341.
2. 100.
3. 13.756^{g}; $.013756^{kg}$.
4. 346.1^{g}.
5. 376.15^{kg}.
6. $677,800^{mg}$.
7. 5800.

EXERCISE IV.

Page 71.

1. 109.59^{m}.
2. $254.12.
3. 57.625^{m}.
4. 3.437^{m}.
5. $11,282^{km}$.
6. 3312^{km}; 2860^{km}; 1600^{km}; 3180^{km}.

7. 5527.9^{km};
6712.45^{km};
$15,478.12^{km}$;
$21,953.66^{km}$;
$288,240.5^{km}$.
8. \$4.11; \$14.15.
9. \$113.63;
\$229.17.
10. \$745.88.
11. \$64.88.
12. \$128.50.
13. \$968.50.
14. \$47.32.

Page 72.

15. 3.1416^{m}.
16. $260,752.8^{mm}$;
$11,655.3^{mm}$;
$103,044.5^{mm}$;
326.7^{mm};
370.7^{mm};
525.0^{mm};
123.5^{mm}.
17. $925,142,191,327^{m}$.
18. $40,005.1344^{km}$.
19. 4.1155^{m};
4.1155^{m};
69.9635^{m}.
20. 16.87014;
17.86248;
4196.33502.
21. .31831.
22. 100^{cm}.
23. 1.7507^{m}.
24. $.94538^{m}$.
25. 636.62^{m}.
26. 6.3662^{cm}.
27. 23.87325^{m}.

EXERCISE V.

Page 73.

1. 323^{qcm}.
2. $11,200^{ha}$;
$11,144.659^{ha}$.
3. 14.222^{ha};
14.166^{ha}.
4. $.169^{ha}$.
5. 15.639^{qm};
33.914^{qm};
40.152^{qm}.

Page 74.

6. 2.49126761^{ha};
2.4909024^{ha};
2.4907176^{ha};
7.47288761^{ha}.
7. \$168.16;
\$168.14;
\$168.12;
\$504.42;
7. \$956.65;
\$956.51;
\$956.44;
\$2869.60;
\$59,167.61;
\$59,158.93;
\$59,154.54;
\$177,481.08.
8. 572.5566^{qcm};
$.7854^{qm}$.
9. 48.275^{ha}.
10. 754.7694^{qcm}.
11. 18^{cm};
36^{m}.
12. 468^{qm};
300^{qm}.

Page 75.

13. 153.9384^{qcm}.
14. 1017.8784^{qcm}.
15. 199.5119^{qm}.
16. 226.1952^{qcm}.

17. 7696.92^{qcm}.
18. 48.275^{ha}.
19. 153.9384^{qcm}.
20. 1.505^{qm}.
21. 1809.5616^{qm}.
22. 5026.56^{qm};
706.86^{qm};
4319.7^{qm}.

Page 76.

24. 99.84^{m}.
25. 54^{m}.
26. 43.76^{m}.
27. 25^{m}; $46.88.
28. 33.25^{m}; $70.66.
29. 44.8^{m}; 40^{cm}; $56.

Page 77.

31. 19.
32. $35.
33. $73.88.
34. $14.
35. $36.14.
36. $19.95.
37. $43.20.

EXERCISE VI.

Page 78.

1. 378^{ccm}. **2.** 73.416^{g}. **3.** 1375.6064^{hl}. **4.** $111,424.1184^{kg}$.

Page 79.

5. $4258.69.
6. 1079.757^{l};
1079.757^{kg}.
7. $445,341^{ccm}$;
445.341^{l};
4.45341^{hl}.
8. 280.68^{hl};
173.086^{hl};
364.884^{hl}.
9. 2.3701^{m};
9.2201^{hl}.
10. 298.452^{ccm}.
11. $.785^{l}$.
12. 4.132^{l};
$.392^{l}$.
13. 523.6^{ccm};
$.0012^{ccm}$.
14. 3811.2^{ccm}, or 3.8112^{l}.

Page 80.

15. 1749.561^{l}, or
17.49561^{hl}.
16. $145,782^{l}$.
17. 223.42 min.
18. 40.459 min.
19. $.903178^{cbm}$.

EXERCISE VII.

1. 100^{kg}; 73.8^{kg};
1000^{kg}; $.001^{kg}$.
2. $5.
3. 264.404^{kg}.
4. 274.980^{kg}.
5. 259.380^{kg}.
6. 955.24^{kg}.

Page 81.

7. 7235^{kg}, or 7.235^{t};
8464.95^{kg}, or 8.46495^{t}.
8. 28.75^{kg}.
9. 24.
10. 363.33^{mg}.

EXERCISE VIII.

Page 82.

1. .833.
2. 678.11^{kg}.
3. 79.249^{kg}; 79.443^{kg}; 79.831^{kg}; 81.286^{kg}; 82.159^{kg}.
4. 64.1806^{g}; 66.6779^{g}.

Page 83.

5. 2.608.
6. 5.1635^{kg}.
7. 12.242^{kg}; 12.215^{kg}.
8. 8.953.
9. 7.309; 283.615^{kg}; 295.765^{kg}.
10. 1.0258; 1.0257.
11. 4.315.
12. 1.723; 2.098.
13. 7.3.
14. 2.317.
15. 2.789.
16. 3.04.

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17. 17.8^{ccm}; 1.893.
18. 13.616; 13.588; 13.592; 13.599.
19. 7.5.

EXERCISE IX.

1. 2.67^{kg}; 18.67^{kg}; 1.14^{kg}; $.38^{kg}$; 2.67^{kg}; 18.67^{kg}; 93.33^{kg}.
2. 161.9^{kg}.
3. 20^{l}; 140^{l}; 3740^{l}.
4. 40^{l}; 13.33^{l}; 560^{l}.
5. 90^{l}; \$4.44.
6. \$0.06; \$0.065.

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7. \$11.50.
8. 7.
9. \$26.25.
10. 11.2 min.
11. 6.588 min.
12. 2.4984 dys.
13. 40 min.
14. 2.533^{l} a min.
15. 265.26.
16. 61.89.
17. 852.6^{kg}.
18. \$159.86.

EXERCISE X.

Page 86.

1. $46{,}609^{m}$.
2. 5013.
3. 840.
4. \$2,876,626.20.
5. \$900,662.50.
6. \$23,062,338; \$7,391,775.
7. 41.145^{m}.
8. 12.16^{m}.
9. $.028^{cm}$.
10. 2304.338^{km}.

Page 87.

11. 6.24^{km}.
12. 19,277.2.
13. 100^{ha};
$10{,}000^{\text{a}}$;
$1{,}000{,}000^{\text{qm}}$.
14. $54{,}200{,}000^{\text{ha}}$.
15. $20,991.60.
16. 716.
17. 1690.
18. $0.135.
19. $300.
20. $.997^2 \times 14.0715^{\text{ha}} = 13.987^{\text{ha}}$.

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21. 121,000;
181.5^{ha}.
22. $.283608^{\text{a}}$.
23. 14.6965^{a}.
24. $.15869^{\text{cbm}}$.
25. 10.547^{m}.
26. 5250^{l}; 2.38^{m}.
27. 3.646^{cbm}.
28. 196.
29. 1.6387^{cbm}.

EXERCISE XI.

Page 93.

1. $2 \times 2 \times 37$;
$2^3 \times 3 \times 11$;
2×89; 3×61;
1×173; 11×17;
2×173; $7 \times 7 \times 7$.
2. $2 \times 3 \times 5 \times 7$;
1×353;
$2^5 \times 3 \times 5 \times 11$;
$3 \times 7 \times 11$;
$2^3 \times 3 \times 7 \times 11 \times 17$;
37×37;
$2^3 \times 3^2 \times 19$.
3. 13×19; 3×109;
1×179; 1×83;
$5^3 \times 17$;
13×181;
1×2333.
4. $3 \times 5 \times 11$;
$2^3 \times 3 \times 7$;
$2^2 \times 3 \times 179$;
$2 \times 3 \times 2777$;
3×107;
$3 \times 11 \times 47$;
2×19.
5. 2×41;
3×43;
$2^3 \times 3^2$;
$2 \times 3 \times 11$;
$2^2 \times 17$;
5×13;
$2^2 \times 19$;
2×43;
$2^3 \times 11$;
2×71.
6. 2×163; $2^4 \times 23$;
$2^4 \times 29$; $2^2 \times 73$;
2×181; 5×73;
$2 \times 5 \times 73$;
$2 \times 3 \times 7$.
7. $2^2 \times 7 \times 31$; $3^3 \times 37$;
$2 \times 3 \times 137$;
2×673; $3^3 \times 283$;
$2 \times 3 \times 1039$;
$2 \times 3^2 \times 13$.
8. 3×193; 1×577;
$2^2 \times 53$; $2 \times 3^2 \times 7$;
2^7; 2^{13};
$2 \times 3^2 \times 5 \times 7 \times 13$.
9. 7×1171;
5^5; 7^4; 11^3;
$3^2 \times 7 \times 11 \times 113$;
$3 \times 7 \times 11 \times 109$.

EXERCISE XII.

1. $2^2 \times 3 \times 7 \times .1$;
$2^3 \times 19 \times .1$;
$2^3 \times 3^3 \times .01$;
$2^4 \times 3^2 \times .001$;
$2^2 \times 3^2 \times .001$; $37 \times .001$;
$3 \times 5 \times 11 \times 13 \times .01$.

2. $2 \times 73 \times .1$;
$3^2 \times 29 \times .01$; $2^3 \times 53 \times .1$;
$2 \times 3 \times 7 \times 11 \times 17 \times .01$;
$2^3 \times 7 \times 11 \times 17 \times .0001$;
$2^3 \times 13 \times .00001$.

3. $3 \times 17^2 \times .01$;
$3 \times 7 \times 23 \times .1$;
$3^2 \times 11 \times 101 \times .01$;
$2^3 \times 3^2 \times 7 \times .01$;
$3^3 \times 5 \times 11 \times .001$;
$2^5 \times 3^3 \times .001$.

4. $11 \times 317 \times .01$;
$2^3 \times 3^4 \times .1$;
$3 \times 5 \times 11 \times 31 \times .001$;
$2^5 \times 89 \times .1$;
$41 \times 73 \times .001$.

EXERCISE XIII.

Page 95.

1. 3.
2. 13.
3. 4.
4. 9.
5. 27.
6. 2.
7. 2.
8. 3.
9. 5.
10. 14.
11. 6.
12. 14.
13. 12.
14. 21.
15. 4.
16. 9.
17. 6.
18. 14.
19. 16.
20. 24.
21. 42.
22. 11.
23. 21.
24. 16.
25. 17.
26. 101.

EXERCISE XIV.

Page 98.

1. 37.
2. 105.
3. 8.
4. 151.
5. 135.
6. 352.
7. 345.
8. 107.
9. 809.
10. 113.
11. 127.
12. 99.
13. 97.
14. 93.
15. 108.
16. 59.
17. 1.
18. 56.
19. 387.
20. 2138.

EXERCISE XV.

Page 99.

1. 57.
2. 23.
3. 247.
4. 327.
5. 107.
6. 449.
7. 37.
8. 119.
9. 111.
10. 227.

EXERCISE XVI.

Page 101.

1. 42.
2. 24.
3. 30.
4. 36.
5. 105.
6. 120.
7. 168.
8. 60.
9. 96.
10. 231.
11. 297.
12. 1001.
13. 385.
14. 432.
15. 660.
16. 7920.
17. 6720.
18. 1296.
19. 1800.
20. 4704.
21. 630.
22. 32,400.
23. 780.
24. 3600.
25. 600.
26. 210.
27. 2520.
28. 2310.
29. 756.
30. 6630.
31. 288.
32. 36,036.
33. 3570.
34. 5040.
35. 27,324.
36. 2340.
37. 27,720.
38. 228,150.
39. 13,800.
40. 2736.
41. 7560.
42. 2400.
43. 32,844.
44. 40,480.
45. 23,400.
46. 576,576.
47. 181,875.
48. 50,592.
49. 29,029.
50. 10,241.

EXERCISE XVII.

Page 102.

1. 4664.
2. 11,803.
3. 141,036.
4. 41,123.
5. 23,069.
6. 27,869.
7. 92,796.
8. 14,385.
9. 5967.
10. 61,318.
11. 7,577,304.
12. 12,397.
13. 164,808.
14. 3,295,313.
15. 260,117.
16. 37,219.
17. 163,601,937.
18. 250,641,677.
19. 34,573.
20. 79,781.

EXERCISE XVIII.

Page 105.

1. $1\frac{6}{7}$.
2. $2\frac{5}{8}$.
3. $6\frac{1}{4}$.
4. $9\frac{8}{11}$.
5. $13\frac{5}{16}$.
6. 11.
7. $14\frac{24}{25}$.
8. 37.
9. $50\frac{73}{75}$.
10. $26\frac{82}{177}$.
11. $3\frac{570}{753}$.
12. 13.
13. $18\frac{7}{29}$.
14. $18\frac{30}{42}$.
15. 359.

EXERCISE XIX.

Page 106.

1. $\frac{31}{9}$.
2. $\frac{59}{10}$.
3. $\frac{136}{11}$.
4. $\frac{197}{19}$.
5. $\frac{59}{7}$.
6. $\frac{193}{15}$.
7. $\frac{1697}{20}$.
8. $\frac{83821}{97}$.
9. $\frac{41083}{1000}$.
10. $\frac{410003}{10000}$.
11. $\frac{160127}{400}$.
12. $\frac{25000083}{5000}$.
13. $\frac{150014}{15}$.
14. $\frac{2100783}{700}$.
15. $\frac{7299}{99}$.

16. $\frac{40}{5}$, $\frac{35}{5}$, $\frac{15}{5}$, $\frac{25}{5}$, $\frac{60}{5}$, $\frac{65}{5}$, $\frac{90}{5}$, $\frac{100}{5}$, $\frac{125}{5}$.
17. $\frac{63}{3}$, $\frac{105}{5}$, $\frac{147}{7}$, $\frac{168}{8}$, $\frac{252}{12}$, $\frac{273}{13}$, $\frac{420}{20}$, $\frac{525}{25}$, $\frac{630}{30}$, $\frac{777}{37}$.
18. $\frac{144}{12}$, $\frac{180}{15}$, $\frac{276}{23}$, $\frac{180}{12}$, $\frac{225}{15}$, $\frac{345}{23}$, $\frac{276}{12}$, $\frac{345}{15}$, $\frac{529}{23}$.

EXERCISE XX.

Page 107.

1. $\frac{5}{8}$.
2. $\frac{7}{9}$.
3. $\frac{116}{165}$.
4. $\frac{12}{17}$.
5. $\frac{16}{81}$.
6. $\frac{3}{4}$.
7. $\frac{11}{14}$.
8. $\frac{5}{16}$.
9. $\frac{8}{13}$.
10. $\frac{11}{13}$.
11. $\frac{8}{9}$.
12. $\frac{27}{91}$.
13. $\frac{17}{23}$.
14. $\frac{1}{4}$.
15. $\frac{144}{175}$.
16. $\frac{7}{80}$.
17. $\frac{8}{15}$.
18. $\frac{35}{54}$.
19. $\frac{12}{49}$.
20. $\frac{32}{767}$.
21. $\frac{97}{117}$.
22. $\frac{212}{315}$.
23. $\frac{349}{481}$.
24. $\frac{3}{11}$.
25. $\frac{11}{25}$.
26. $\frac{35}{47}$.
27. $\frac{79}{91}$.
28. $\frac{293832}{349921}$.
29. $\frac{65}{999}$.
30. $\frac{3}{7}$.

EXERCISE XXI.

Page 108.

1. $1\frac{1}{2}$.
2. $6\frac{3}{4}$.
3. 4.
4. 15.
5. 3.
6. 10.
7. $1\frac{1}{4}$.
8. $\frac{2}{3}$.
9. 15.
10. $1\frac{3}{16}$.
11. $1\frac{19}{20}$.
12. $2\frac{3}{5}$.
13. $3\frac{1}{4}$.
14. $3\frac{9}{10}$.
15. $4\frac{11}{20}$.
16. $5\frac{1}{5}$.
17. $6\frac{1}{2}$.
18. $7\frac{4}{5}$.
19. $9\frac{3}{4}$.
20. 13.

EXERCISE XXII.

Page 109.

1. $\frac{14}{33}$.
2. $\frac{9}{10}$.
3. $\frac{5}{21}$.
4. 6.
5. $10\frac{2}{7}$.
6. $45\frac{1}{9}$.
7. 2.
8. $\frac{1}{3}$.
9. $1\frac{1}{5}$.
10. $3\frac{3}{4}$.
11. $1\frac{7}{20}$.
12. $48\frac{3}{10}$.
13. $1\frac{1}{3}$.
14. $\frac{5}{6}$.
15. $1\frac{178}{627}$.
16. $\frac{117}{946}$.
17. $8\frac{2}{11}$.
18. $1\frac{1}{3}$.
19. 1.

Page 110.

20. $6\frac{6}{55}$.
21. $\frac{51}{710}$.
22. $\frac{3}{4}$.
23. $\frac{1}{15}$.
24. 8.
25. $\frac{3}{4}$.
26. $2\frac{13}{16}$.
27. $52\frac{4}{27}$.
28. $20\frac{5}{11}$.
29. $32\frac{1}{12}$.
30. $\frac{28}{45}$.
31. $\frac{9}{76}$.
32. $\frac{32}{405}$.
33. $\frac{50}{243}$.

EXERCISE XXIII.

1. $\frac{4}{35}$.
2. $\frac{2}{11}$.
3. $\frac{3}{56}$.
4. $2\frac{2}{3}$.
5. $\frac{5}{6}$.
6. 2.
7. $\frac{21}{40}$.
8. $1\frac{4}{35}$.
9. 2.
10. $1\frac{17}{25}$.
11. $\frac{27}{38}$.
12. $1\frac{6}{7}$.
13. $7\frac{1}{2}$.
14. $\frac{9}{14}$.
15. $1\frac{1}{6}$.
16. 3.
17. $\frac{9}{14}$.
18. 2.
19. $6\frac{2}{7}$.

EXERCISE XXIV.

Page 112.

1. $\frac{15\ \ 12\ \ 25}{30}$.
2. $\frac{240\ \ 200\ \ 315\ \ 324}{360}$.
3. $\frac{700\ \ 105\ \ 200\ \ 456}{840}$.
4. $\frac{120\ \ 315\ \ 108\ \ 160}{900}$.
5. $\frac{288\ \ 255\ \ 180\ \ 152}{600}$.
6. $\frac{315\ \ 196\ \ 96\ \ 90\ \ 665}{840}$.
7. $\frac{1485\ \ 840\ \ 1404\ \ 1656\ \ 680}{2160}$.
8. $\frac{17}{25}$, $\frac{5}{6}$, $\frac{3}{5}$.
9. $\frac{13}{24}$, $\frac{7}{12}$, $\frac{11}{18}$.
10. $\frac{4}{11}$, $\frac{7}{18}$, $\frac{5}{12}$, $\frac{8}{15}$.

EXERCISE XXV.

Page 114.

1. 2.
2. $1\frac{1}{3}$.
3. $1\frac{1}{4}$.
4. 4.
5. 4.
6. 4.
7. $6\frac{2}{5}$.
8. $2\frac{1}{4}$.
9. $2\frac{3}{17}$.
10. $21\frac{3}{17}$.
11. $1\frac{19}{30}$.
12. $1\frac{5}{8}$.
13. $\frac{2}{3}$.
14. $\frac{49}{60}$.
15. $\frac{37}{48}$.
16. $19\frac{13}{16}$.
17. $113\frac{7}{36}$.
18. $1\frac{17}{60}$.
19. $2\frac{43}{60}$.
20. $3\frac{1}{15}$.
21. $85\frac{83}{600}$.
22. $199\frac{47}{63}$.
23. $12\frac{1}{10}$.
24. $8\frac{73}{100}$.
25. $11\frac{263}{504}$.
26. $11\frac{461}{504}$.
27. $10\frac{733}{840}$.
28. $1\frac{179}{200}$.
29. $14\frac{61}{72}$.
30. $18\frac{9}{176}$.
31. $8\frac{13}{18}$.
32. $10\frac{31}{77}$.
33. $341\frac{107}{120}$.
34. $\frac{173}{228}$.
35. $1\frac{35}{204}$.
36. $346\frac{401}{510}$.
37. $29\frac{36}{385}$.
38. $181\frac{241}{2880}$.
39. $294\frac{1069}{1092}$.
40. $465\frac{411}{616}$.

EXERCISE XXVI.

Page 116.

1. $6\frac{1}{3}$.
2. $\frac{1}{3}$.
3. $\frac{1}{12}$.
4. $\frac{7}{60}$.
5. $\frac{25}{63}$.
6. $3\frac{1}{2}$.
7. $6\frac{1}{3}$.
8. $2\frac{1}{6}$.
9. $7\frac{4}{7}$.
10. $4\frac{1}{5}$.
11. $4\frac{2}{9}$.
12. $1\frac{1}{6}$.
13. $\frac{34}{35}$.
14. $5\frac{1}{30}$.
15. $2\frac{23}{45}$.
16. $3\frac{11}{12}$.
17. $4\frac{29}{30}$.
18. $4\frac{1}{6}$.
19. $2\frac{1}{12}$.
20. $4\frac{3}{4}$.
21. $3\frac{22}{35}$.
22. $57\frac{70}{99}$.
23. $6\frac{1}{30}$.
24. $6\frac{3}{8}$.
25. $10\frac{11}{96}$.
26. $4\frac{117}{140}$.
27. $10\frac{323}{440}$.
28. $\frac{749}{960}$.
29. $2\frac{1683}{4000}$.
30. $1193\frac{1}{12}$.
31. $1193\frac{73}{156}$.
32. $1193\frac{17}{36}$.
33. $248\frac{25}{48}$.
34. $114\frac{47}{66}$.
35. $101\frac{37}{154}$.
36. $2\frac{31}{154}$.
37. $4\frac{19}{42}$.
38. $\frac{48}{119}$.
39. $\frac{1}{756}$.
40. $\frac{13}{418}$.
41. $\frac{218}{1911}$.
42. $\frac{1}{450}$.

EXERCISE XXVII.

Page 118.

1. $\frac{20}{33}$.
2. $\frac{8}{19}$.
3. $1\frac{2}{7}$.
4. $\frac{1}{15}$.
5. $\frac{11}{18}$.
6. $1\frac{11}{21}$.
7. $4\frac{8}{15}$.
8. $2\frac{82}{567}$.
9. $1\frac{2}{7}$.
10. $1\frac{1}{2}$.
11. $10\frac{1}{35}$.
12. $\frac{1}{24}$.
13. $\frac{3}{7}$.
14. $\frac{2}{5}$.
15. $3\frac{1}{2}$.
16. $2\frac{29}{30}$.
17. 185.
18. $\frac{53}{122}$.
19. $\frac{8}{9}$.
20. 20.
21. $\frac{9}{35}$.
22. 1.

EXERCISE XXVIII.

Page 119.

1. $\frac{3}{8}$. 8. $3\frac{3}{8}$. 14. $\frac{3}{4}$. 20. $\frac{7}{8}$. 26. $\frac{1}{7}$. 32. 1.
2. $2\frac{2}{3}$. 9. $\frac{5}{11}$. 15. 2. 21. $\frac{1}{15}$. 27. $1\frac{1}{7}$. 33. $\frac{1}{135}$.
3. $\frac{7}{9}$. 10. $2\frac{1}{5}$. 16. $\frac{1}{8}$. 22. $\frac{3}{50}$. 28. $\frac{4}{15}$. 34. $2\frac{16}{33}$.
4. $1\frac{2}{7}$. 11. $3\frac{3}{56}$. 17. $\frac{3}{10}$. 23. $\frac{9}{200}$. 29. $110\frac{375}{578}$. 35. $\frac{32627}{32825}$.
5. $1\frac{1}{2}$. 12. $\frac{51}{176}$. 18. $\frac{4}{33}$. 24. $6\frac{1}{4}$. 30. $\frac{9}{10}$. 36. $1\frac{142}{7047}$.
6. $\frac{2}{3}$. 13. $2\frac{31}{70}$. 19. $\frac{1}{5}$. 25. $\frac{76}{905}$. 31. 1. 37. $\frac{7}{9}$.
7. $\frac{3}{11}$. 38. $\frac{102}{113}$.

EXERCISE XXIX.

Page 120.

1. $\frac{1}{8}$. 6. $3\frac{1}{4000}$. 11. $10\frac{503}{40000}$. 16. $20\frac{147}{8000}$. 21. $\frac{193}{320}$.
2. $\frac{5}{8}$. 7. $8\frac{43}{400}$. 12. $104\frac{47}{200}$. 17. $125\frac{373}{625}$. 22. $6\frac{1}{32}$.
3. $\frac{27}{40}$. 8. $35\frac{82}{3125}$. 13. $50\frac{1}{2500}$. 18. $\frac{16}{125}$. 23. $60\frac{5}{16}$.
4. $10\frac{108}{125}$. 9. $7\frac{1}{64}$. 14. $100\frac{1}{1000}$. 19. $\frac{117}{160}$. 24. $7\frac{63}{2000}$.
5. $50\frac{21}{25}$. 10. $20\frac{3133}{31250}$. 15. $8\frac{29}{4000}$. 20. $1\frac{3}{16}$.

EXERCISE XXX.

1. .875. 6. 4.01375. 11. .00425. 16. 7.75.
2. .9375. 7. 5.00015625. 12. .92969. 17. 1.2.
3. .28125. 8. 9.00480. 13. .0208. 18. .328125.
4. .36. 9. 11.00475. 14. .04297. 19. 14.8.
5. .078125. 10. .072. 15. .01875. 20. .69385.

EXERCISE XXXI.

Page 124.

1. 132 in. 2. 23,166 ft. 3. 5499 in. 4. 5,294,820 ft.
5. 1309 mi. 192 rds. 4 yds. 2 ft. 8 in. 6. 443,646 in.
7. 1,754,676 in. 8. 45,112,320 in.
9. 102 mi. 114 rds. 3 yds. 1 ft.
10. 4 mi. 89 rds. 5 yds. 1 ft. 2 in.
11. 137 mi. 36 rds. 3 yds. 1 ft.
12. 3 mi. 217 rds. 4 yds. 1 ft. 3 in.
13. 7920 in. 14. 3238 fath. 4 ft. 1 in.
15. 5 ft. 4 in. 16. $27\frac{3}{11}$ mi.

Exercise XXXII.

Page 125.

1. 120,058,848 sq. in. **4.** 1148 sq. rds. 2 sq. yds.
2. 944 sq. yds. 103 sq. in. **5.** 23,048,771 sq. in.
3. 73,830,400 sq. rds. **6.** 43,560 sq. ft.
7. 15 A. 148 sq. rds. 2 sq. yds. 2 sq. ft. 40 sq. in.
8. 3 A. 28 sq. rds. 9 sq. yds.
9. 14 sq. rds. 10 sq. yds. 7 sq. ft. 110 sq. in.

Exercise XXXIII.

Page 126.

1. 202 cu. ft. **4.** 9 cu. yds. 11 cu. ft. 88 cu. in.
2. 1,175,183 cu. in. **5.** 19 cu. yds. 20 cu. ft. 706 cu. in.
3. 43 cu. ft. 21 cu. in. **6.** $9\frac{1}{7}$ ft. **7.** $101.86.

Exercise XXXIV.

Page 127.

1. 59 pts. **4.** 233 qts. $1\frac{1}{2}$ pts. **7.** .8036 bu.
2. 66 bu. 5 qts. **5.** 465 qts. 0 pts. $3\frac{7}{11}$ gi. **8.** 109,792 pts.
3. 790 gi. **6.** 1.244 cu. ft. **9.** 96 bbls. 23 gals.

Exercise XXXV.

Page 128.

1. 4 lbs. 9 oz. 9 dwts. 11 grs. **5.** 123 lbs. 7 oz. 5.2 drs.
2. 15 l. t. 9 l. cwt. 44 lbs. **6.** 32,316 grs.
3. 4 t. 5 cwt. 53 lbs. 3 oz. **7.** 905 lbs. 4 oz. 11 dwt. 16 grs.
4. 150 lbs. 8 dwts. 13 grs. **8.** 613 lbs. $7\frac{11}{35}$ drs.
9. 45 t. 27 lbs. 13 oz. 5 drs.
10. 176 t. 5 cwt. 48 lbs. 5 oz. 13 drs.
11. 7680 ♏. **12.** 16 fl. oz. 5 fl. drms. 20 ♏.

Exercise XXXVI.

Page 129.

1. 22,645 sec. **4.** 8 dys. 14 hrs. 13 min. 12 sec.
2. 544,324 min. **5.** 151; 118; 286; 151 dys.
3. 33 dys. 17 hrs. 27 min. **6.** 151,200 min.
7. 468,000 sec. **8.** 1600, 1656, 1880, 1924, 2000.

Exercise XXXVII.

Page 130.

1. 9025″. **2.** 54,202″. **3.** 15° 46′. **4.** 58° 57′ 1″.
5. 90°, 120°, (180°), 45°, 30°, 0°.
6. 2815 geog. mi.; 3244.75$\frac{2}{3}$ stat. mi.

Exercise XXXVIII.

Page 131.

1. 140,000 *d.* **7.** £303 10*s.* 3 *d.* 2 far.
2. 76,750 far. **8.** 2813 *g.* 20 *s.* 10 *d.*
3. £119 1 *s.* **9.** 1193 crowns 6 *d.* 3 far.
4. 2268 half sov. 2 *s.* 7*d.* **10.** $3.86.
5. 6495 *g.* 10 *s.* **11.** $4.76.
6. £1,801,914 14 *s.* **12.** $3.67.
13. £61 2 *s.* 6 *d.*

Exercise XXXIX.

Page 133.

1. 15° C.; 12° R. **8.** − 22° F.; − 24° R.
2. 25° C.; 20° R. **9.** 11$\frac{3}{7}$° F.; − 9$\frac{1}{7}$° R.
3. 510° C.; 408° R. **10.** 41° F.; 5° C.
4. − 40° C.; − 32° R. **11.** 59° F.; 15° C.
5. − 20° C.; − 16° R. **12.** − 13° F.; − 25° C.
6. 50° F.; 8° R. **13.** 39$\frac{1}{5}$° F.; 3$\frac{1}{5}$° R.
7. 71$\frac{3}{5}$° F.; 17$\frac{3}{5}$° R. **14.** − 17$\frac{7}{9}$° C.; − 14$\frac{2}{9}$° R.

Exercise XL.

Page 135.

1. 3 dys. 4 hrs. 34 min. 36 sec. **3.** £92 12 *s.* 1 *d.* 3 far.
2. 245 cu. yds. 4 cu. ft. 1514 cu. in. **4.** 35 mi. 126 rds. 4 yds. 1 in.
5. 151 A. 55 sq. rds. 29 sq. yds. 7 sq. ft. 34 sq. in.
6. 23 bu. 6 qts. 1 pt. **9.** 84 lbs. 2 oz. 14 dwt.
7. 212 t. 6 cwt. 3 oz. 10 drs. **10.** 41 gals. 1 pt.
8. 243 gals. 1 qt. 1 gi. **11.** 132° 30′ 45″.

EXERCISE XLI.

Page 136.

1. 34 lbs. 9 oz. 18 dwt. 11 grs. **3.** 7 cu. yds. 3 cu. ft. 1528 cu. in.
2. 1 pk. 4 qts. 1 pt. **4.** £27 18 *s.* 2¾ *d.*
5. 7 gals. 2 qts. 1 pt.
6. 8 t. 5 cwt. 98 lbs. 5 oz. 11 drs.
7. 6 A. 9 sq. rds. 174 sq. ft. 26 sq. in.
8. 31° 37′ 45″. **9.** 298 dys. 21 hrs. **10.** 202 rds. 4 yds. 3 in.

EXERCISE XLII.

Page 137.

1. £249 4 *d.* **3.** 33 lbs. 8 oz. 4 dwt. 4 grs.
2. 1391 gals. 1 qt. **4.** 55 t. 6 cwt. 70 lbs.
5. 2740 bu. 2 pks.

Page 138.

6. 108 wks. 1 dy. 12 hrs. 24 min.
7. 32 cu. yds. 7 cu. ft. 498 cu. in.
8. 3 sq. mi. 553 A. 27 sq. rds. 3 sq. yds. 2 sq. ft. 36 in.
9. 5 mi. 258 rds. 3 yds.
10. 10 yrs. 67 dys. 23 hrs. 45 min. 36 sec.
11. 38,508 mi. 19 rds. 3 yds.
12. 2147 lbs. 9 ℥ 4 ʒ 1 ℈ 7 grs.

EXERCISE XLIII.

Page 139.

1. 1 mi. 207 rds. 2 yds. 8 in. **2.** 137.
3. 8 wks. 1 dy. 7 hrs. 50 min. $2\frac{121}{848}$ sec.
4. 77. **5.** 325. **6.** 7 sq. mi. 264 A. 39 sq. rds. **7.** 7.

EXERCISE XLIV.

Page 140.

1. 256 rds. **3.** 62 lbs. 8 oz.
2. 30 sq. rds. **4.** 13 *s.* 4 *d.*
5. 261 rds. 4 yds. 1 ft. 6 in.
6. 101 sq. rds. 24 sq. yds. 6 sq. ft. 108 sq. in.

7. 26′ 40″.
8. 121 dys. 16 hrs.
9. 5 qts.
10. 2 qts. 1 pt.
11. 320 dys. 6 hrs.
12. 3 oz. 18 dwt.

EXERCISE XLV.

Page 141.

1. 23 A. 40 sq. rds. 16 sq. yds. 1 sq. ft. $28\frac{4}{5}$ sq. in.
2. 10 hrs. 30 min. $38\frac{4}{7}$ sec.
3. 15 dys. 10 hrs. 38 min. 51 sec.
4. 34 rds. 2 yds. $4\frac{4}{15}$ in.
5. 57 rds. 2 yds. 2 ft. 7.5936 in.
6. 5 t. 6 cwt. 94 lbs.
7. 8 oz. 6 dwt. $2\frac{2}{3}$ grs.
8. 1 lb. 6 oz. 13 dwt. 8 grs.
9. 3 mi. 198 rds. 4 yds. $4\frac{1}{2}$ in.
10. 2 wks. 2 dys. 3 hrs. 35 min.
11. 12 dys. 10 hrs. 31 min. 30 sec.
12. 86 rds. 4 yds. $1\frac{1}{2}$ in.
13. 4 cwt. 72 lbs. 4 oz. $9\frac{1}{7}$ drs.

EXERCISE XLVI.

Page 142.

1. $\frac{175}{144}$. **4.** $\frac{2}{3}$. **7.** $\frac{5}{48}$. **10.** $\frac{1}{6}$. **13.** $\frac{59}{8640}$. **16.** $\frac{78}{125}$.
2. $\frac{175}{192}$. **5.** $\frac{40}{63}$. **8.** $\frac{7}{185}$. **11.** $\frac{1}{20}$. **14.** $\frac{21}{64}$. **17.** $\frac{42161}{174240}$.
3. $\frac{3}{40}$ A. **6.** $\frac{1}{70}$. **9.** $\frac{28}{45}$. **12.** $\frac{3}{11}$. **15.** $\frac{887}{400}$.

EXERCISE XLVII.

Page 143.

1. £0.815625. **3.** .445 A. **5.** .242. **7.** .0311.
2. .73 mi. **4.** .3174. **6.** .501. **8.** .303.

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9. $.\dot{1}4\dot{8}$. **10.** .243.
11. 5.128 (reckoning 365 days).
12. 1.139. **13.** .15433. **14.** .03. **15.** .4388.

EXERCISE XLVIII.

Page 145.

1. 18° 45′.
2. 32° 45′.
3. 77° 32′ 30″.
4. 51° 23′ 45″.
5. 93° 7′ 30″.
6. 62° 3′.
7. 272° 30′.
8. 228° 48′ 45″.
9. 37 min. 20 sec.
10. 4 hrs. 42 min.
11. 3 hrs. 46 min. $24\frac{4}{5}$ sec.
12. 7 hrs. 14 min. $10\frac{2}{5}$ sec.
13. 8 hrs. 58 sec.
14. 6 hrs. 43 min. 3.6 sec.
15. 8 min. $8\frac{2}{15}$ sec.
16. 5 hrs. 40 sec.

EXERCISE XLIX.

Page 147.

1. $43\frac{1}{5}$ mi.
2. 10,800 mi.
3. 4655.637 mi.
4. $1173\frac{1}{3}$ ft.
5. $4\frac{4}{23}$ mi.

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6. $4\frac{29}{64}$ mi.
7. $6\frac{9}{94}$ in.
8. $2\frac{4}{5}$ min.
9. $2\frac{1}{6}$ min.
10. $4\frac{52}{75}$ sec.
11. 1 min. $43\frac{88}{189}$ sec.
12. $\frac{7}{15}$ day; $\frac{3}{14}$ A.
13. 1 A. 120 sq. rds.
14. $156\frac{2}{3}$ gals.
15. $5035.20.
16. 12 hrs.

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17. $3922.20.
18. $1\frac{22}{149}$ pts.
19. $1040\frac{140}{359}$ mi.
20. 24 mi.; 1 hr. 44 min. 10 sec.
21. $1\frac{5}{7}$ dys.
22. $2\frac{19}{40}$ dys.

Page 150.

23. 1 hr. 58 min. $7\frac{1}{2}$ sec.
24. $5\frac{23}{247}$ hrs.
25. $1\frac{7}{29}$ dys.
26. $2\frac{16}{19}$ dys.
27. $\frac{6}{13}$ hr.
28. $56\frac{1}{4}$ min.; 125 gals.
29. 1 hr. 42 min. $51\frac{3}{7}$ sec.
30. 1 hr. 3 min. $34\frac{62}{67}$ sec.
31. $2\frac{22}{159}$ all. $4\frac{24}{79}$, A; $5\frac{55}{57}$, B; $14\frac{13}{23}$, C.

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32. 992 bu.
33. 18 girls; 32 scholars.
34. 26 boys; 22 girls.
35. 35.
36. 120.
37. 35.
38. 18 dwt. 8 grs. troy; $1\frac{1}{175}$ oz. av.
39. A, $21,000; B, $15,000; C, $9000.
40. $1\frac{5}{13}$.
41. $13,200.
42. $\frac{16}{27}$, $\frac{15}{26}$, $\frac{13}{25}$.

Page 152.

43. \$0.75. **46.** $19\frac{1278}{1900}$ lbs. **49.** 27 min. **52.** $1\frac{1}{2}$ dys.
44. \$2665. **47.** \$1.44. **50.** \$9.336. **53.** 324.
45. \$25.42. **48.** $\frac{14}{165}$. **51.** $61\frac{1}{3}$ A. **54.** 90.

Page 153.

55. 300. **56.** 7 mos. **57.** $34\frac{284}{455}$. **58.** 16. **59.** 500.

Page 154.

60. 20. **62.** 4 dys. **64.** 2 dys. **66.** $6\frac{3}{7}$ yds. **68.** $1396\frac{4}{11}$.
61. 11,268. **63.** 84. **65.** 15. **67.** $2\frac{2}{5}$ ft. **69.** $67\frac{2}{3}$.
70. C the fastest; A 41 sec., B $19\frac{1}{15}$ sec.

Page 155.

71. \$89.40. **72.** \$7573.36. **73.** \$1830.70.

EXERCISE L.

Page 156.

1. 9 ft. **3.** 13 A. 5.792 sq. ch. **5.** 418 yds. = 76 rds.
2. 60 rds. or 990 ft. **4.** 3 ft. **6.** 12 sq. ft.

Page 157.

7. 5 sq. ft. 21 sq. in. **10.** 27,648. **13.** $344\frac{4}{9}$ sq. yds. border;
8. 36. **11.** 2 sq. ft. 81 sq. in. $411\frac{1}{9}$ sq. yds. path;
9. 15. **12.** \$15.20. 3500 sq. yds. grass.
14. 28.2744 sq. ft. **16.** 16,060.6446 sq. ft.
15. 502,656 sq. yds. **17.** 132.7326 sq. ft.

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18. 258.5248 sq. ft. **22.** $60\frac{2}{3}$ yds.; 63 yds.; none; $\frac{1}{3}$ yd.
19. 28.2744 sq. in. **23.** $59\frac{15}{16}$ yds. or 60 yds.
20. 452.3904 sq. in. **24.** $17\frac{1}{2}$ sq. yds.
21. 4 sq. ft. 52.32 sq. in. **25.** \$43.70.

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26. \$45.00 or \$46.67. **29.** Lengthwise. **32.** \$46.50.
27. \$123.375. **30.** $81\frac{1}{2}$ sq. yds. **33.** \$4.26.
28. \$194.79. **31.** $199\frac{67}{324}$ sq. yds.

Page 160.

34. \$40.67. **35.** \$12.75. **36.** \$33.75. **37.** \$62.50.

Page 161.

38. $13\frac{1}{2}$; **40.** 78. **43.** 33. **46.** 325. **49.** \$9.94. **52.** \$32.84.
$14\frac{2}{3}$. **41.** 162. **44.** 140. **47.** \$2.36. **50.** \$1.80. **53.** \$10.71.
39. 233. **42.** 207. **45.** 300. **48.** \$14.85. **51.** \$3.92.

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54. 187. **56.** 302. **58.** \$0.80. **60.** \$1.38.
55. 83. **57.** 123. **59.** \$1.22. **61.** \$0.79.

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62. 810. **63.** $3\frac{6}{25}$ in. **64.** $24\frac{1}{2}$ thousand.

EXERCISE LI.

Page 163.

1. 16 cu. ft. 72 cu. in. **3.** 15 cu. yds. 16 cu. ft. 1512 cu. in.
2. 38,016 cu. ft. **4.** $120\frac{3}{4}$ cu. yds. **5.** $52\frac{5}{8}$ cu. ft.

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6. 63 cu. in. **8.** \$855. **10.** 2 ft. 8 in. **12.** 259,200.
7. 1512. **9.** $1\frac{5}{6}$ yds. **11.** $13\frac{14}{47}$ ft. **13.** $58\frac{1}{3}$ ft.
14. 72 sq. ft. $48\frac{2}{3}$ sq. in. **16.** 14,088.9375 or $14{,}088\frac{15}{16}$ t.
15. 36 sq. ft. 36 sq. in. **17.** 14,400.

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18. $\$194.79\frac{1}{6}$. **20.** $45\frac{5}{7}$ lbs. **22.** 8 ft. **24.** $7\frac{11}{35}$ ft.
19. $13\frac{1}{2}$ ft. **21.** $6\frac{2}{3}$. **23.** \$13.67. **25.** $2\frac{2}{3}$ ft.

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26. $76\frac{4}{5}$ bu. **29.** $16\frac{2}{3}$ ft. **31.** \$1440. **33.** $5\frac{32}{35}$ t.
27. 156 bu. **30.** 107.1 bu. **32.** $26\frac{1}{4}$. **34.** $4278\frac{6}{7}$ gals.
28. $6\frac{1}{4}$ ft.

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35. $132\frac{76}{77}$ gals. **39.** 4.90875 gals. **43.** .267.
36. $7\frac{37}{77}$ gals. **40.** 2.18 qts. **44.** 28.51.
37. 840 cu. ft. **41.** 696.9116 cu. in. **45.** .859.
38. 1484.406 gals. **42.** 15.708.

EXERCISE LII.

Page 169.

1. 6 : 9. **2.** 9 : 12. **3.** 8 : 9. **4.** 8 : 14. **5.** \$7 : \$9.
6. 8 ft. : 11 ft. **7.** 5 : 3. **8.** $\frac{5}{8}$ yd. : $\frac{6}{8}$ yd.

EXERCISE LIII.

Page 172.

1. 16 dys.
2. $11\frac{7}{10}$.
3. 2320.
4. $5.89.
5. $40\frac{1}{2}$.

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6. $2\frac{1}{12}$ sq. ft.
7. $2\frac{19}{28}$ hrs.
8. 17 ft. $1\frac{1}{14}$ in.
9. 139 ft. $11\frac{65}{101}$ in.
10. 8.
11. $1\frac{5}{7}$ hrs.
12. $39\frac{13}{33}$ sec.
13. $34\frac{6}{191}$ sec.
14. 9 min. $59\frac{1}{2}$ sec.
15. 5 wks. 3 dys. 21 hrs. 20 min.
16. 2673.
17. $7435.78.
18. 1 t. 4 cwt. $45\frac{1}{2}$ lbs.
19. $32\frac{1}{7}$.

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20. .8 lb.
21. 2565 francs.
22. 2306.85 lbs.
23. $4165.
24. 13 yds.
25. $89\frac{103}{193}$ cu. in.
26. $8\frac{19}{22}$ mo.
27. $1\frac{59}{76}$ in.
28. $\frac{5}{16}$ in.
29. 50.
30. 21,000.
31. $2\frac{26}{37}$.

EXERCISE LIV.

Page 176.

1. $7\frac{1}{2}$.
2. $26.44.
3. $23,356.67.
4. 124 lbs.
5. 1368 bu.
6. 14 dys.
7. 540.

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8. 6027.
9. $1.575.
10. 56,875 gals.
11. $188\frac{16}{23}$ yds.
12. 3 hrs.
13. $10\frac{1}{2}$.
14. 38 ft. 1.5 in.
15. 150 yds.
16. $5\frac{29}{47}$ in.
17. 60^{m}.
18. 1134.2776^{kg}.
19. 246.4 cu. ft.
20. 3.8 cu. ft.

EXERCISE LV.

Page 179.

1. $3000, $4000, $5000.
2. 150 t., 200 t., 225 t., 240 t.
3. 1155 lbs., 1848 lbs., 3850 lbs.;
3560 lbs., 2225 lbs., 1068 lbs.
4. $2000; $1600.
5. 700 lbs.; 21 lbs.
6. $730\frac{43}{50}$ lbs.; $206\frac{7}{50}$ lbs.
7. $37,500^{kg}$ saltpetre,
6250^{kg} charcoal,
6250^{kg} sulphur.
8. $10\frac{2}{3}$.
9. 746.46; 210.54.

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10. $66\frac{2}{3}$ lbs., $33\frac{1}{3}$ lbs.
11. 23.203^{g} oxygen, 76.797^{g} nitrogen.
12. $45.60.

EXERCISE LVI.

1. $\$1333.33\frac{1}{3}$, A's; $\$666.66\frac{2}{3}$, B's.
2. $\$863.28\frac{1}{8}$, $\$836.71\frac{7}{8}$.
3. $948, A's; $752, B's; $816, C's.

Page 181.

4. $710, A's; $1196.25, B's.
5. $882, A's; $759.50, B's; $525, C's.
6. $37.50; $60; $35.
7. $656.50, A's; $286.50, B's.
8. $1837.50, A's; $3311, B's.
9. £36 15*s.*; £14 14*s.*; £14.
10. $1811.25, A's; $1794, B's.

EXERCISE LVII.

Page 184.

1. 13,257.
2. 25.55 t.
3. 1500 lbs. saltpetre, 200 lbs. sulphur, 300 lbs. charcoal.
4. 19.2 t.
5. 350.46 cu. ft.
6. 6052.
7. 660.
8. 12%.

Page 185.

9. 66.
10. $20\frac{40}{83}\%$.
11. $3\frac{51}{83}\%$.
12. $12,500.
13. $1\frac{9}{16}\%$ loss.
14. $\frac{3}{196}$ of 1%.
15. $6\frac{6}{7}$ t.
16. $5\frac{15}{47}\%$.
17. $2750.
18. 8%.
19. 15%.
20. $7\frac{11}{27}\%$ water, $92\frac{16}{27}\%$ alcohol.
21. $216.
22. $13\frac{41}{48}\%$.
23. $9\frac{3}{8}\%$ gain.

Page 186.

24. $\$3.93\frac{3}{4}$.
25. $7\frac{11}{80}\%$ gain.
26. $\$4.16\frac{2}{3}$.
27. $6\frac{2}{3}\%$.
28. $14\frac{2}{7}$ oz.
29. $7.50.
30. 5.4 t. silver, 714.6 t. lead.
31. 3038.936 t.

Pages 187–188.

32. 30%.
33. $28\frac{4}{7}\%$.
34. $35\frac{5}{17}\%$.
35. 100%.
36. $5\frac{5}{19}\%$.
37. 3,510,000; 13:87.
38. $14\frac{2}{7}\%$; $12\frac{1}{2}\%$.
39. $6400; $7200.
40. 30 cents; $26\frac{2}{3}$ cents.
41. $143.
42. $99\frac{21}{100}\%$.
43. $2\frac{19}{28}\%$.
44. $2\frac{14}{23}\%$.
45. $2\frac{19}{28}\%$.
46. $16\frac{2}{3}\%$.
47. 25%.
48. $1\frac{1}{2}$ lbs.
49. .00125.
50. $\frac{1}{700}$; $87\frac{1}{2}$ gals. oxygen, 100 gals nitrogen.
51. .00129.
52. $96\frac{116}{129}$.
53. $\frac{7}{74}$ of 1%.
54. $5\frac{31}{37}$ in.

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55. $492\frac{17}{37}$ ft.

EXERCISE LVIII.

Page 190.

1. $13.06. **4.** $40; 2560 bu. **9.** $210.
2. $64.88. **5.** $5047.64. **7.** $12,445.31. **10.** $6955.56.
3. $1611.30. **6.** $1\frac{1}{2}$%. **8.** $869.05. **11.** $1.523.

Page 191.

12. $1\frac{621}{700}$%. **15.** $13,770. **17.** $14,159.25.
13. 6758.6. **16.** $141. **18.** $58,536\frac{24}{41}$ lbs.
14. 37,951.2 lbs.; $128.05. **19.** $195.07; 141.51562 t.

EXERCISE LIX.

Page 192.

1. $13.25. **2.** $56.25. **3.** $1001. **4.** $12,266.67; $214.67.

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5. $12,000. **8.** $6.25. **11.** $8425. **13.** $96.
6. $1040. **9.** $363.75. **12.** $97,272. **14.** $1840.
7. $1836. **10.** (1) $1500; (2) $750; (3) $1125; (4) $1125.

EXERCISE LX.

Page 194.

1. $96.79. **2.** $962,326.53. **3.** $3.866. **4.** $15,306.12.

Page 195.

5. $6. **7.** $204.80. **9.** $4032. **11.** $1240.70.
6. $17,818.91. **8.** $1587.60. **10.** $28\frac{4}{5}$%. **12.** $4869.12.
13. $1781.80.

EXERCISE LXI.

Page 197.

1. $95.94. **2.** $0.13. **3.** $8.46.

Page 198.

4. $183.33. **9.** $7.56. **14.** $68.62.
5. $327.25. **10.** $3.88. **15.** $9.46.
6. $1.60. **11.** $51.16. **16.** $28.18.
7. $12.91. **12.** $62.04. **17.** $54.27.
8. $13.19. **13.** $37.45. **18.** $16.69.

19. $50.60.
20. $10.29.
21. $483.28.
22. $487.83.
23. $323.35.
24. $6980.12.
25. $166.98.
26. $548.22.
27. $1268.40.
28. $660.44.
29. $12,474.55.
30. $11,481.45.

Page 200.

31. $4\frac{1}{2}$%.
32. $4\frac{1}{2}$%.
33. $4\frac{1}{8}$%.
34. 6%.
35. 4%, nearly.
36. $3\frac{3}{4}$%.
37. $4\frac{1}{2}$%.
38. $4\frac{24}{125}$%.
39. 4%.
40. 6%, almost.
41. $7\frac{1}{7}$%.
42. $7\frac{23}{81}$%.
43. 4%.
44. 4 yrs.
45. 2 yrs.
46. 6 yrs., almost.
47. 3 mos. 9 dys.
48. 25 yrs.

Page 201.

49. 11 yrs. 8 mos.
50. 3 yrs. 4 mos.
51. 3 yrs. 9 mos.
52. 3 mos. 2 dys.
53. 3 yrs. 5 mos.
54. $750.
55. $336.
56. $235.29.
57. $8520.
58. $411.43.
59. $1754.39.
60. $8423.14
or $8432.
61. $9723.33.
62. $4090.91.

Page 202.

63. $750.
64. $816.896.
65. $5982.55.
66. $286.15.
67. $6050.99.
68. $19,600.
69. $166.
70. $560.
71. $8432.
72. $8520.
73. 27.53.
74. 7%, nearly.
75. 1 mo. 8 dys.
76. $524.10.
77. $22,222.22.
78. 2 yrs. 10 mos. 3 dys.
79. $6\frac{28}{37}$%.
80. $47.57.
81. 3 yrs. 4 mos. 24 dys.
82. $4.44.
83. $10.20.

EXERCISE LXII.

Page 205.

1. May 1/4, 112 dys., $16.33; $733.67.
2. Mar. 4/7, 60 dys., $46.86; $4278.64.
3. Apr. 6/9, 73 dys., $19.03; $1321.67.
4. Apr. 19/22, 2 mos. 21 dys., $16.38; $1439.92.
5. June 2/5, 3 mos. 19 dys., $75.78; $4474.58.

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6. Apr. 4/7, 3 mos. 6 dys., $110.46; $5067.46.
7. Apr. 1/4, 48 dys., $47.60; $4712.40.
8. Apr. 14/17, 1 mo. 16 dys., $25.78; $1992.07.
9. June 25/28, 3 mos. 13 dys., $8.61; $660.48.
10. Mar. 20/23, 35 dys., $48.34; $8991.66.
11. Feb. 27/Mar. 2, 27 dys., $0.96¾; $214.03¼.

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12. $865.65.
13. $1630.64.
14. $1207.19.
15. $4056.80.
16. $4558.23.
17. $1120.26.

Page 208.

18. $478.09.
19. $3059.28; $275.34.
20. $3909.48; $351.85.
21. $2348.02; $68.48.
22. $640.
23. $512.65; $61.52.
24. $596.12; $29.01.
25. $683.31; $31.89.

Page 209.

26. $390.31.

Page 210.

27. $162.31.
28. $283.71.
29. $234.13.

Page 212.

30. $1547.73.
31. $391.01.
32. $525.58.
33. $293.39.
34. $17.61.

Page 214.

35. $433.02.
36. $703.31.
37. $195.69.
38. $3.52.
39. $24.73.
40. $40.62.
41. $303.77.
42. $125.
43. $1562.50.
44. $1441.47.
45. $1078.43.

Exercise LXIII.

Page 216.

1. $4395.
2. $2450.
3. $3046.87½.
4. $4723.50.
5. $1096.87½.
6. $11,600.
7. $4650.
8. $5200.
9. $5000.
10. 24.
11. 97½.

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12. $91\frac{5}{8}$.
13. $1188.
14. $270.
15. $329.
16. $30,000.
17. $9800.
18. $754.
19. $3\frac{378}{492}\%$.
20. 9%.
21. $857.14.
22. $271.19.

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23. $366.15.
24. $390.
25. $3\frac{19}{47}\%$.
26. $5\frac{1}{8}\%$.
27. $5\frac{5}{7}\%$.
28. $5\frac{5}{17}\%$.
29. $6\frac{8}{57}\%$.
30. $4\frac{8}{13}\%$.
31. $5\frac{5}{7}\%$.
32. $5635.
33. $4925.
34. $5671.
35. $83\frac{1}{3}$.

Exercise LXIV.

Page 222.

1. 54.
2. 89.
3. 703.
4. 142.
5. 1829.
6. 354.2.
7. 6879.
8. 147.
9. 7.33.
10. 29.72.
11. 1.4045.
12. .01446.
13. 1.414213.
14. 2.236067
15. .547722.
16. 1.802775.
17. 2.972092.
18. .948683.
19. 311.13.
20. 12 ft. 3 in.
21. 8 ft. 11 in.
22. .666666; .745355; .707106; .774596; .845154; .866025; .816496; .912870.
23. 45; 4.2; 1.42; 23.1; 14.3.

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24. 5 ft.
25. 91 in.
26. 60 in.
27. 25 ft.
28. 9 ft.

Exercise LXV.

Page 228.

1. 11.
2. 12.
3. 2.3.
4. 6.7.
5. 53.
6. 127.
7. 3.91.
8. 67.31.
9. .1985.
10. 4.7117.
11. 25,805.
12. 49.68.
13. 2.1544; 1.5361; 2.0274.
14. 1.7099; .8220; 1.9660; .9085.
15. 35 sq. ft. 75.84 sq. in.

Exercise LXVI.

Page 230.

1. 1 : 16; 1 : 64.
2. 8 : 9261 : 10,648,000.
3. 1 : 2; 1 : 4.
4. 1 : 2.121.
5. 1 : 2.154.
6. 1 : .687.

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7. 8.299 ft.
8. 5040 sq. in.
9. 10,648 cu. in.
10. 174.1 lbs.; 6 ft. 4.6 in.
11. 1:.693.
12. 9:16; 27:64.
13. 3.931 in.
14. 12 ft. 7.19 in.
15. 150 rds.
16. 1:.63.

MISCELLANEOUS PROBLEMS.

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1. $57\frac{3}{8}g$.
2. $139\frac{13}{70}$ lbs.
3. $1.02; $0.34.
4. $$0.33\frac{1}{6}$.
5. $$0.30\frac{16}{21}$.
6. $0.47.
7. 1.256.
8. 1.171.
9. 1:3.988.
10. 4:5.
11. $1102.75.
12. $1346.63.
13. $1595.20.
14. $491.38.
15. $1184.30.
16. $946.08.
17. $716.85.
18. $800.52.
19. 123, 132, 213, 231, 312, 321; 1332.
21. P. 200; C. 101; B. 189.
22. 196.
23. 1052.09 sq. in.
24. 1533.623 ft.
25. 5268.585 ft.
26. 808.97 ft.
27. 29.67 ft.
28. 55.31 ft.
29. 12,480.
30. $15.
31. $17\frac{1}{2}$ min.
32. $12\frac{1}{2}$ min.
33. 1367 sq. ft.
34. $.1467^{ha}$.
35. .8908 A.
36. 224.25 A.
37. 25.12 sq. rds.
38. 3.946 A.
39. .1942.
40. 7.407 A.
41. .6436 A.
42. 866.0254 ft.
43. 13.2^{mm}; 17.62^{mm}; 23.36^{mm}.
44. 61,785 sq. ft.
45. 25, 949, 225, 1225, 441, 11025.
46. 92 ft.
47. 7974 dys.
48. 78.54 sq. in.; 376.99 sq. in.; 534.07 sq. in.; 4.08 gals.
49. 8935.
50. 5.424 in.
51. 31.952 cu. in.
52. 50.93 bu.
53. $59{,}259\frac{7}{27}$.
54. $66{,}966\frac{2}{3}$.
55. $.14067^{l}$.
56. 98.47^{cm}.
57. 1.833^{l}.
58. 112.07^{l}.
59. 3.302.
60. .796 pt.; 2.489 pts.; 3.825 pts.
61. 22.286.
62. 69.115 sq. in.; 247.401 sq. in.
63. 11 in.; 13.603; 117.5 sq. in.

64. 9.784 in.; 2.446 in.

65. 8.17.

66. 1,000,000^{kg}.

67. 9,216,000 lbs.

68. 6135 times the weight of the material.

69. 5.453 times its weight when on a level with the hand; 4.453 times when highest; 6.453 times when lowest.

70. 7.355^{m}; 31.869^{m}.

71. 21.04.

72. 23.66.

73. 28.5.

74. 1950^{m}.

75. 8475 ft.

76. 53.88 mi.

77. A, 70.5 lbs.; B, 45.5 lbs.

78. 51$\frac{3}{7}$ min.

79. 2240 lbs.; 2400 lbs.

80. 9 ft.; 7 ft.

81. 17.68^{kg}.